Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.6-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [78]. This is test number [17].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (78)	% 0. (0)
Mathematica	% 100. (78)	% 0. (0)
Maple	% 100. (78)	% 0. (0)
Maxima	% 24.36 (19)	% 75.64 (59)
Fricas	% 55.13 (43)	% 44.87 (35)
Sympy	% 19.23 (15)	% 80.77 (63)
Giac	% 44.87 (35)	% 55.13 (43)

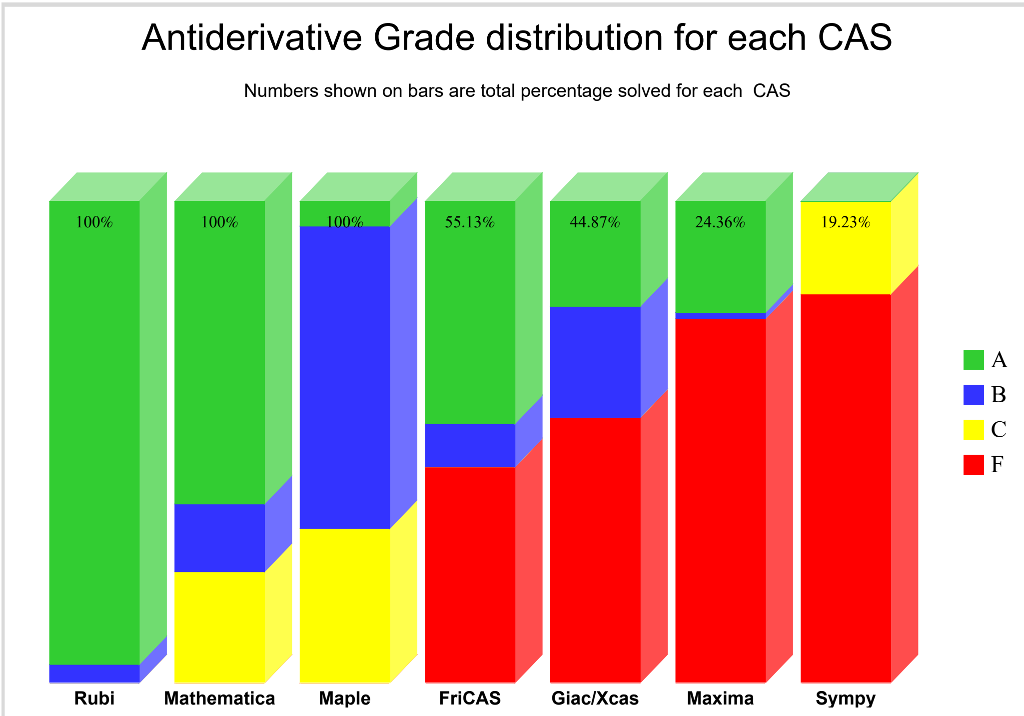
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

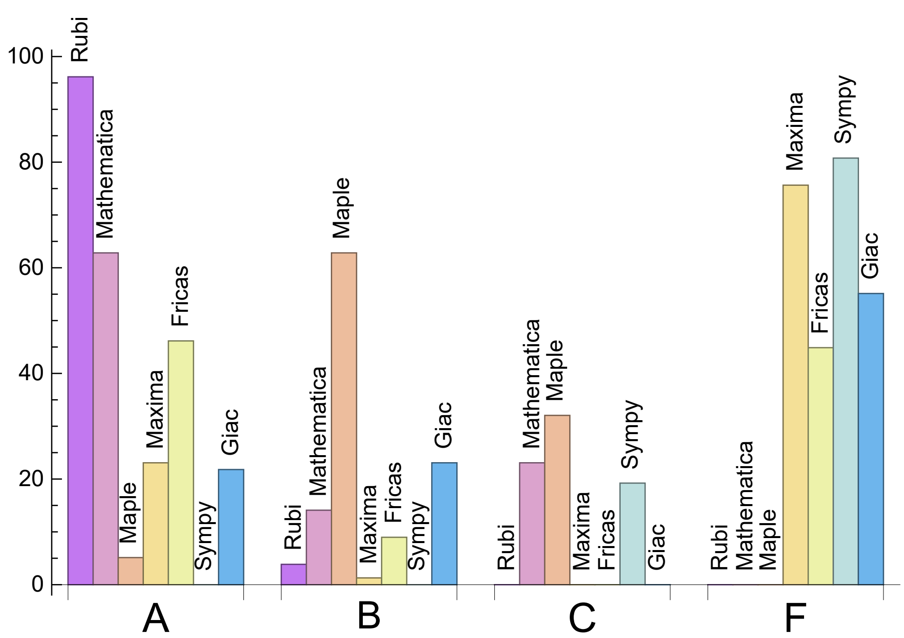
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	96.15	3.85	0.	0.
Mathematica	62.82	14.1	23.08	0.
Maple	5.13	62.82	32.05	0.
Maxima	23.08	1.28	0.	75.64
Fricas	46.15	8.97	0.	44.87
Sympy	0.	0.	19.23	80.77
Giac	21.79	23.08	0.	55.13

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.05	438.29	1.08	350.5	1.
Mathematica	4.41	1428.1	2.02	367.	1.1
Maple	0.05	5530.69	7.88	1315.5	3.9
Maxima	3.44	206.68	1.64	134.	1.6
Fricas	9.98	1302.3	4.59	879.	3.35
Sympy	44.55	314.93	3.8	277.	3.96
Giac	5.09	1199.54	3.09	603.	2.29

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {34, 35, 36, 37, 40, 45, 46, 47, 53, 60, 65, 66, 72, 78}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

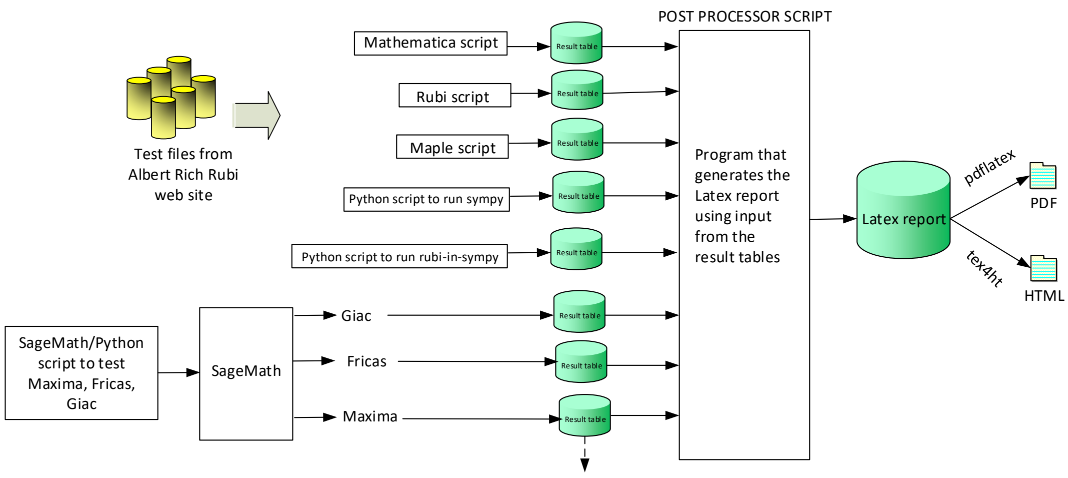
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade: { 35, 36, 37 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 43, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60 }

B grade: { 27, 35, 36, 41, 42, 44, 45, 46, 47, 48, 54 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F grade: { }

2.1.3 Maple

A grade: { 23, 28, 29, 30 }

B grade: { 20, 21, 22, 24, 25, 26, 27, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 34, 35, 36, 37, 38, 39 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 34, 36, 37, 38, 39

B grade: { 35 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 54, 55, 56 }

B grade: { 5, 6, 7, 12, 13, 14, 40 }

C grade: { }

F grade: { 24, 25, 26, 31, 32, 33, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { 10, 11, 15, 16, 17, 18, 19, 29, 30, 34, 35, 36, 37, 38, 39 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.7 Giac

A grade: { 4, 8, 9, 10, 11, 15, 16, 34, 35, 36, 37, 47, 48, 49, 54, 55, 56 }

B grade: { 1, 2, 3, 26, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 57, 58 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 52, 53, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	355	959	644	888	0	1122
normalized size	1	1.	0.86	2.31	1.55	2.14	0.	2.7
time (sec)	N/A	0.673	0.494	0.029	1.994	1.1	0.	3.147

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	244	652	459	617	0	772
normalized size	1	1.	0.85	2.28	1.6	2.16	0.	2.7
time (sec)	N/A	0.563	0.326	0.013	3.756	1.104	0.	1.654

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	170	141	377	263	386	0	429
normalized size	1	1.01	0.84	2.24	1.57	2.3	0.	2.55
time (sec)	N/A	0.25	0.17	0.01	3.295	1.086	0.	3.044

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	185	154	224	0	198
normalized size	1	1.	0.75	1.95	1.62	2.36	0.	2.08
time (sec)	N/A	0.073	0.062	0.011	4.046	1.042	0.	1.896

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	117	373	0	1019	0	0
normalized size	1	1.	0.96	3.06	0.	8.35	0.	0.
time (sec)	N/A	0.311	0.145	0.04	0.	29.882	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	211	899	0	2082	0	0
normalized size	1	1.	1.29	5.52	0.	12.77	0.	0.
time (sec)	N/A	0.331	0.442	0.041	0.	118.982	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	273	1449	0	3105	0	0
normalized size	1	1.	1.1	5.84	0.	12.52	0.	0.
time (sec)	N/A	0.355	0.396	0.045	0.	1.448	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	241	643	524	644	0	551
normalized size	1	1.	0.71	1.89	1.54	1.89	0.	1.62
time (sec)	N/A	0.633	0.37	0.025	3.199	1.143	0.	2.271

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	160	423	356	435	0	352
normalized size	1	1.	0.7	1.86	1.56	1.91	0.	1.54
time (sec)	N/A	0.493	0.207	0.024	4.342	1.143	0.	2.824

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	133	88	235	205	267	617	186
normalized size	1	1.02	0.68	1.81	1.58	2.05	4.75	1.43
time (sec)	N/A	0.23	0.101	0.018	3.604	1.087	112.876	1.983

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	105	167	282	97
normalized size	1	1.	0.71	1.86	1.67	2.65	4.48	1.54
time (sec)	N/A	0.061	0.034	0.016	3.776	1.019	20.572	3.103

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	117	373	0	1019	0	0
normalized size	1	1.	0.96	3.06	0.	8.35	0.	0.
time (sec)	N/A	0.283	0.127	0.	0.	30.013	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	211	899	0	2082	0	0
normalized size	1	1.	1.29	5.52	0.	12.77	0.	0.
time (sec)	N/A	0.295	0.413	0.	0.	117.112	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	273	1449	0	3105	0	0
normalized size	1	1.	1.1	5.84	0.	12.52	0.	0.
time (sec)	N/A	0.329	0.179	0.	0.	1.43	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	139	134	189	313	123
normalized size	1	1.	0.72	1.76	1.7	2.39	3.96	1.56
time (sec)	N/A	0.139	0.061	0.	4.966	1.142	46.387	2.264

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	105	167	282	97
normalized size	1	1.	0.71	1.86	1.67	2.65	4.48	1.54
time (sec)	N/A	0.061	0.032	0.	2.397	1.043	20.862	1.866

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	96	89	196	245	0
normalized size	1	1.	1.	2.	1.85	4.08	5.1	0.
time (sec)	N/A	0.183	0.052	0.	4.226	1.183	28.239	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	97	89	201	221	0
normalized size	1	1.	1.	2.02	1.85	4.19	4.6	0.
time (sec)	N/A	0.176	0.056	0.	3.266	1.147	27.753	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	108	132	154	218	0
normalized size	1	1.	0.79	1.52	1.86	2.17	3.07	0.
time (sec)	N/A	0.184	0.047	0.	3.974	1.018	34.289	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	584	427	1446	0	2147	0	0
normalized size	1	0.99	0.72	2.45	0.	3.63	0.	0.
time (sec)	N/A	1.517	1.408	0.038	0.	1.45	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	450	311	987	0	1517	0	0
normalized size	1	1.	0.69	2.19	0.	3.36	0.	0.
time (sec)	N/A	1.01	0.995	0.017	0.	1.307	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	297	200	588	0	980	0	0
normalized size	1	0.99	0.67	1.96	0.	3.27	0.	0.
time (sec)	N/A	0.446	0.646	0.013	0.	1.274	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	142	287	0	602	0	0
normalized size	1	1.	0.64	1.3	0.	2.72	0.	0.
time (sec)	N/A	0.147	0.38	0.011	0.	1.148	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	225	503	0	0	0	0
normalized size	1	1.	0.81	1.81	0.	0.	0.	0.
time (sec)	N/A	0.49	0.786	0.054	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	309	1200	0	0	0	0
normalized size	1	1.	0.96	3.73	0.	0.	0.	0.
time (sec)	N/A	0.579	1.033	0.046	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	361	492	1848	0	0	0	2238
normalized size	1	0.99	1.36	5.09	0.	0.	0.	6.17
time (sec)	N/A	0.677	1.916	0.052	0.	0.	0.	16.905

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	496	1107	965	0	1538	0	0
normalized size	1	0.99	2.21	1.93	0.	3.07	0.	0.
time (sec)	N/A	1.281	6.542	0.028	0.	2.118	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	369	555	635	0	1065	0	0
normalized size	1	1.	1.51	1.73	0.	2.89	0.	0.
time (sec)	N/A	0.875	3.814	0.025	0.	1.913	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	249	390	365	0	689	736	0
normalized size	1	1.01	1.59	1.48	0.	2.8	2.99	0.
time (sec)	N/A	0.4	1.624	0.022	0.	1.701	138.497	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	169	180	0	460	338	0
normalized size	1	1.	0.95	1.02	0.	2.6	1.91	0.
time (sec)	N/A	0.124	0.414	0.018	0.	1.643	25.875	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	225	503	0	0	0	0
normalized size	1	1.	0.81	1.81	0.	0.	0.	0.
time (sec)	N/A	0.464	0.729	0.	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	309	1200	0	0	0	0
normalized size	1	1.	0.96	3.73	0.	0.	0.	0.
time (sec)	N/A	0.53	0.969	0.	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	361	492	1848	0	0	0	2238
normalized size	1	0.99	1.36	5.09	0.	0.	0.	6.17
time (sec)	N/A	0.588	1.836	0.	0.	0.	0.	10.678

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	151	149	137	147	176	308	130
normalized size	1	1.74	1.71	1.57	1.69	2.02	3.54	1.49
time (sec)	N/A	0.146	0.344	0.	2.139	1.624	44.26	2.21

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	C	B	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	135	126	120	142	150	277	104
normalized size	1	2.6	2.42	2.31	2.73	2.88	5.33	2.
time (sec)	N/A	0.071	0.214	0.	2.84	1.633	21.362	2.518

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	135	128	95	86	184	240	96
normalized size	1	2.45	2.33	1.73	1.56	3.35	4.36	1.75
time (sec)	N/A	0.185	0.407	0.	2.306	1.56	26.976	2.134

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	135	89	96	86	203	216	112
normalized size	1	2.45	1.62	1.75	1.56	3.69	3.93	2.04
time (sec)	N/A	0.18	0.167	0.	3.561	1.1	28.173	2.621

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	129	82	103	88	173	212	196
normalized size	1	1.55	0.99	1.24	1.06	2.08	2.55	2.36
time (sec)	N/A	0.191	0.117	0.	4.02	1.173	33.628	1.987

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	171	94	123	119	216	219	266
normalized size	1	1.47	0.81	1.06	1.03	1.86	1.89	2.29
time (sec)	N/A	0.217	0.116	0.	3.643	1.031	58.436	2.363

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	242	336	1095	0	2485	0	817
normalized size	1	1.22	1.69	5.5	0.	12.49	0.	4.11
time (sec)	N/A	0.328	0.813	0.052	0.	1.248	0.	3.459

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1348	1345	3599	6728	0	6765	0	3560
normalized size	1	1.	2.67	4.99	0.	5.02	0.	2.64
time (sec)	N/A	2.366	7.098	0.046	0.	10.902	0.	3.682

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	719	2722	3571	0	3633	0	2006
normalized size	1	1.	3.78	4.95	0.	5.04	0.	2.78
time (sec)	N/A	0.963	6.483	0.021	0.	3.786	0.	3.573

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	306	1431	0	1843	0	856
normalized size	1	1.	0.93	4.34	0.	5.58	0.	2.59
time (sec)	N/A	0.298	1.849	0.016	0.	1.543	0.	1.583

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	453	1944	4227	0	0	0	1461
normalized size	1	1.01	4.32	9.39	0.	0.	0.	3.25
time (sec)	N/A	1.369	6.199	0.044	0.	0.	0.	3.323

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	2665	5051	0	0	0	2140
normalized size	1	1.	5.12	9.69	0.	0.	0.	4.11
time (sec)	N/A	1.696	6.274	0.042	0.	0.	0.	13.147

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	658	657	2157	12065	0	0	0	11268
normalized size	1	1.	3.28	18.34	0.	0.	0.	17.12
time (sec)	N/A	2.68	6.449	0.063	0.	0.	0.	38.844

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1032	1032	3220	3958	0	4766	0	2032
normalized size	1	1.	3.12	3.84	0.	4.62	0.	1.97
time (sec)	N/A	1.788	6.672	0.042	0.	43.625	0.	1.977

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	2402	2002	0	2503	0	994
normalized size	1	1.	4.45	3.71	0.	4.64	0.	1.84
time (sec)	N/A	0.713	6.327	0.026	0.	9.366	0.	1.391

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	225	763	0	1277	0	425
normalized size	1	1.	0.91	3.1	0.	5.19	0.	1.73
time (sec)	N/A	0.23	1.07	0.018	0.	2.511	0.	2.662

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	465	1822	0	0	0	797
normalized size	1	1.	1.6	6.28	0.	0.	0.	2.75
time (sec)	N/A	0.672	3.86	0.033	0.	0.	0.	1.809

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	417	3670	0	0	0	1874
normalized size	1	1.	1.15	10.08	0.	0.	0.	5.15
time (sec)	N/A	1.097	2.924	0.042	0.	0.	0.	12.504

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	535	9100	0	0	0	0
normalized size	1	1.	1.11	18.8	0.	0.	0.	0.
time (sec)	N/A	1.563	6.305	0.088	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	685	685	739	15990	0	0	0	0
normalized size	1	1.	1.08	23.34	0.	0.	0.	0.
time (sec)	N/A	1.778	6.331	0.148	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	715	2195	2528	0	3170	0	1284
normalized size	1	1.	3.06	3.52	0.	4.42	0.	1.79
time (sec)	N/A	1.336	6.518	0.04	0.	14.866	0.	4.73

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	369	379	1199	0	1631	0	603
normalized size	1	0.99	1.02	3.23	0.	4.4	0.	1.63
time (sec)	N/A	0.509	2.018	0.03	0.	4.981	0.	3.439

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	173	425	0	879	0	262
normalized size	1	1.	1.05	2.59	0.	5.36	0.	1.6
time (sec)	N/A	0.149	0.766	0.02	0.	2.514	0.	3.686

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	304	746	0	0	0	467
normalized size	1	1.	1.62	3.97	0.	0.	0.	2.48
time (sec)	N/A	0.341	1.	0.031	0.	0.	0.	1.632

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	324	2973	0	0	0	1831
normalized size	1	1.	1.28	11.7	0.	0.	0.	7.21
time (sec)	N/A	0.638	2.011	0.052	0.	0.	0.	11.154

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	513	7119	0	0	0	0
normalized size	1	1.	1.21	16.79	0.	0.	0.	0.
time (sec)	N/A	0.967	2.734	0.107	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	826	826	800	18802	0	0	0	0
normalized size	1	1.	0.97	22.76	0.	0.	0.	0.
time (sec)	N/A	2.433	6.081	0.246	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1182	1154	11933	14778	0	0	0	0
normalized size	1	0.98	10.1	12.5	0.	0.	0.	0.
time (sec)	N/A	4.166	17.717	0.088	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	769	917	10271	0	0	0	0
normalized size	1	0.99	1.18	13.27	0.	0.	0.	0.
time (sec)	N/A	2.23	13.329	0.042	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	706	706	633	6257	0	0	0	0
normalized size	1	1.	0.9	8.86	0.	0.	0.	0.
time (sec)	N/A	1.845	8.114	0.051	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	687	938	16177	0	0	0	0
normalized size	1	1.	1.37	23.55	0.	0.	0.	0.
time (sec)	N/A	1.903	13.409	0.096	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	964	964	9529	34395	0	0	0	0
normalized size	1	1.	9.88	35.68	0.	0.	0.	0.
time (sec)	N/A	3.116	16.847	0.209	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1716	1716	15719	68345	0	0	0	0
normalized size	1	1.	9.16	39.83	0.	0.	0.	0.
time (sec)	N/A	7.045	19.426	0.36	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1235	1235	12483	15857	0	0	0	0
normalized size	1	1.	10.11	12.84	0.	0.	0.	0.
time (sec)	N/A	4.395	18.448	0.056	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	766	766	922	9544	0	0	0	0
normalized size	1	1.	1.2	12.46	0.	0.	0.	0.
time (sec)	N/A	2.061	13.015	0.043	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	562	6049	0	0	0	0
normalized size	1	1.	1.07	11.48	0.	0.	0.	0.
time (sec)	N/A	0.98	9.696	0.032	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	551	4732	0	0	0	0
normalized size	1	1.	1.02	8.76	0.	0.	0.	0.
time (sec)	N/A	1.111	6.814	0.043	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	596	724	13614	0	0	0	0
normalized size	1	1.	1.21	22.8	0.	0.	0.	0.
time (sec)	N/A	1.359	11.974	0.094	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1034	1034	9186	33007	0	0	0	0
normalized size	1	1.	8.88	31.92	0.	0.	0.	0.
time (sec)	N/A	3.16	16.589	0.213	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	838	831	1000	10546	0	0	0	0
normalized size	1	0.99	1.19	12.58	0.	0.	0.	0.
time (sec)	N/A	2.167	13.841	0.051	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	524	615	6174	0	0	0	0
normalized size	1	0.99	1.16	11.69	0.	0.	0.	0.
time (sec)	N/A	1.028	8.052	0.035	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	384	418	2497	0	0	0	0
normalized size	1	0.99	1.08	6.45	0.	0.	0.	0.
time (sec)	N/A	0.505	5.836	0.028	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	477	3984	0	0	0	0
normalized size	1	1.	1.13	9.44	0.	0.	0.	0.
time (sec)	N/A	0.691	5.566	0.045	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	699	12981	0	0	0	0
normalized size	1	1.	1.09	20.22	0.	0.	0.	0.
time (sec)	N/A	1.517	10.931	0.122	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1116	1116	8844	34102	0	0	0	0
normalized size	1	1.	7.92	30.56	0.	0.	0.	0.
time (sec)	N/A	3.342	16.579	0.297	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [36] had the largest ratio of [0.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.	37	0.162
2	A	6	6	1.	37	0.162
3	A	5	5	1.01	35	0.143
4	A	5	5	1.	30	0.167
5	A	6	6	1.	37	0.162
6	A	6	6	1.	37	0.162
7	A	5	5	1.	37	0.135
8	A	6	5	1.	37	0.135
9	A	5	5	1.	37	0.135
10	A	4	4	1.02	35	0.114
11	A	4	4	1.	30	0.133
12	A	6	6	1.	37	0.162
13	A	6	6	1.	37	0.162
14	A	5	5	1.	37	0.135
15	A	4	4	1.	31	0.129
16	A	4	4	1.	30	0.133
17	A	7	7	1.	33	0.212
18	A	7	7	1.	33	0.212
19	A	6	6	1.	33	0.182
20	A	8	7	0.99	40	0.175
21	A	7	7	1.	40	0.175
22	A	6	6	0.99	38	0.158
23	A	6	6	1.	33	0.182
24	A	7	7	1.	40	0.175
25	A	7	7	1.	40	0.175
26	A	5	5	0.99	40	0.125
27	A	7	6	0.99	40	0.15
28	A	6	6	1.	40	0.15
29	A	5	5	1.01	38	0.132
30	A	5	5	1.	33	0.152
31	A	7	7	1.	40	0.175
32	A	7	7	1.	40	0.175
33	A	5	5	0.99	40	0.125
34	A	5	5	1.74	30	0.167
35	B	5	5	2.6	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	B	8	8	2.45	32	0.25
37	B	8	8	2.45	32	0.25
38	A	6	6	1.55	32	0.188
39	A	7	7	1.47	32	0.219
40	A	5	5	1.22	32	0.156
41	A	8	7	1.	36	0.194
42	A	7	6	1.	34	0.176
43	A	7	6	1.	29	0.207
44	A	9	8	1.01	36	0.222
45	A	9	8	1.	36	0.222
46	A	9	9	1.	36	0.25
47	A	7	7	1.	36	0.194
48	A	6	6	1.	34	0.176
49	A	6	6	1.	29	0.207
50	A	8	8	1.	36	0.222
51	A	8	8	1.	36	0.222
52	A	8	8	1.	36	0.222
53	A	6	6	1.	36	0.167
54	A	6	6	1.	36	0.167
55	A	5	5	0.99	34	0.147
56	A	5	5	1.	29	0.172
57	A	7	7	1.	36	0.194
58	A	7	7	1.	36	0.194
59	A	5	5	1.	36	0.139
60	A	6	5	1.	36	0.139
61	A	10	7	0.98	38	0.184
62	A	9	7	0.99	38	0.184
63	A	9	7	1.	38	0.184
64	A	9	8	1.	38	0.21
65	A	9	7	1.	38	0.184
66	A	10	8	1.	38	0.21
67	A	10	7	1.	38	0.184
68	A	9	7	1.	38	0.184
69	A	8	7	1.	38	0.184
70	A	8	7	1.	38	0.184
71	A	8	7	1.	38	0.184
72	A	9	8	1.	38	0.21
73	A	9	7	0.99	38	0.184
74	A	8	7	0.99	38	0.184
75	A	7	6	0.99	38	0.158
76	A	7	6	1.	38	0.158
77	A	8	7	1.	38	0.184
78	A	9	7	1.	38	0.184

Chapter 3

Listing of integrals

3.1 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=415

$$-\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}+\frac{\left(1-d^2x^2\right)^{3/2}\left(3d^2fx\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf\right.\right.}{}$$

```
[Out] ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B
*f^3)*x*Sqrt[1 - d^2*x^2])/(16*d^4) - ((7*d^2*f*(B*e + 2*A*f) - C*(3*d^2*e^
2 - 8*f^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(70*d^4*f) + ((3*C*e - 7*B*f)*
(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(42*d^2*f) - (C*(e + f*x)^4*(1 - d^2*x^2)^
(3/2))/(7*d^2*f) + ((8*(C*(3*d^4*e^4 - 30*d^2*e^2*f^2 - 8*f^4) - 7*d^2*f*(2
*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2))) + 3*d^2*f*(6*C*d^2*e^3 - 1
4*B*d^2*e^2*f - 41*C*e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x)*(1 - d^2*x^2)^(3
/2))/(840*d^6*f) + ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2
+ 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(16*d^5)
```

Rubi [A] time = 0.672762, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 833, 780, 195, 216}

$$-\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}+\frac{\left(1-d^2x^2\right)^{3/2}\left(3d^2fx\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B
*f^3)*x*Sqrt[1 - d^2*x^2])/(16*d^4) - ((7*d^2*f*(B*e + 2*A*f) - C*(3*d^2*e^
2 - 8*f^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(70*d^4*f) + ((3*C*e - 7*B*f)*
(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(42*d^2*f) - (C*(e + f*x)^4*(1 - d^2*x^2)^
(3/2))/(7*d^2*f) + ((8*(C*(3*d^4*e^4 - 30*d^2*e^2*f^2 - 8*f^4) - 7*d^2*f*(2
*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2))) + 3*d^2*f*(6*C*d^2*e^3 - 1
4*B*d^2*e^2*f - 41*C*e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x)*(1 - d^2*x^2)^(3
/2))/(840*d^6*f) + ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2
+ 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(16*d^5)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)dx = \int (e+fx)^3\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} - \frac{\int (e+fx)^3\left(-\left(4C+7Ad^2\right)f^2+d^2f(\cdots}{7d^2f^2}$$

$$= \frac{(3Ce-7Bf)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} + \cdots$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{70d^4f} + \cdots$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{70d^4f} + \cdots$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-\cdots}}{16d^4}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-\cdots}}{16d^4}$$

Mathematica [A] time = 0.493917, size = 355, normalized size = 0.86

$$\frac{\sqrt{1-d^2x^2}\left(14Ad^2\left(6d^4x\left(20e^2fx+10e^3+15ef^2x^2+4f^3x^3\right)-d^2f\left(120e^2+45efx+8f^2x^2\right)-16f^3\right)+7B\left(4d^6x^2\left(45\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*x
^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3*d
^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x
^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) - C*(128
*f^3 + d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) + 6*d^4*x*(35*e^3 + 56*e^2*
f*x + 35*e*f^2*x^2 + 8*f^3*x^3) - 12*d^6*x^3*(35*e^3 + 84*e^2*f*x + 70*e*f^
2*x^2 + 20*f^3*x^3))) + 105*d*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f +
3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(1680*d^6)
```

Maple [C] time = 0.029, size = 959, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/1680*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(630*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^
(1/2))*d^3*e*f^2+630*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e^2*f+315
*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*e*f^2-560*B*csgn(d)*(-d^2*x^2+1
)^(1/2)*d^4*e^3-224*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f^3+240*C*csgn(d)*x^6*
d^6*f^3*(-d^2*x^2+1)^(1/2)+280*B*csgn(d)*x^5*d^6*f^3*(-d^2*x^2+1)^(1/2)+336
*A*csgn(d)*x^4*d^6*f^3*(-d^2*x^2+1)^(1/2)+420*C*csgn(d)*x^3*d^6*e^3*(-d^2*x
^2+1)^(1/2)+560*B*csgn(d)*x^2*d^6*e^3*(-d^2*x^2+1)^(1/2)-48*C*csgn(d)*(-d^2
*x^2+1)^(1/2)*x^4*d^4*f^3-70*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x^3*d^4*f^3-112*A
```

```
*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*f^3-1680*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d
^4*e^2*f-64*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^2*f^3-672*B*csgn(d)*(-d^2*x^
2+1)^(1/2)*d^2*e*f^2-672*C*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*e^2*f+840*A*csgn(
d)*(-d^2*x^2+1)^(1/2)*x*d^6*e^3-210*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e^3-
105*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f^3+840*A*arctan(csgn(d)*d*x/(-d^2*x
^2+1)^(1/2))*d^5*e^3-128*C*csgn(d)*(-d^2*x^2+1)^(1/2)*f^3+210*C*arctan(csgn
(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e^3+105*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(
1/2))*d*f^3-630*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e*f^2-630*B*csgn(d)*(-d^
2*x^2+1)^(1/2)*x*d^4*e^2*f-315*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*e*f^2+840
*C*csgn(d)*x^5*d^6*e*f^2*(-d^2*x^2+1)^(1/2)+1008*B*csgn(d)*x^4*d^6*e*f^2*(-
d^2*x^2+1)^(1/2)+1008*C*csgn(d)*x^4*d^6*e^2*f*(-d^2*x^2+1)^(1/2)+1260*A*csg
n(d)*x^3*d^6*e*f^2*(-d^2*x^2+1)^(1/2)+1260*B*csgn(d)*x^3*d^6*e^2*f*(-d^2*x^
2+1)^(1/2)+1680*A*csgn(d)*x^2*d^6*e^2*f*(-d^2*x^2+1)^(1/2)-210*C*csgn(d)*(-
d^2*x^2+1)^(1/2)*x^3*d^4*e*f^2-336*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*e*f
^2-336*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*e^2*f)*csgn(d)/d^6/(-d^2*x^2+1)
^(1/2)
```

Maxima [A] time = 1.99422, size = 644, normalized size = 1.55

$$-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cf^3x^4}{7\,d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}Ae^3x+\frac{Ae^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Be^3}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Ae^2f}{d^2}-\frac{4\left(-d^2x^2+1\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^3*x +
1/2*A*e^3*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^
3/d^2 - (-d^2*x^2 + 1)^(3/2)*A*e^2*f/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*
x^2/d^4 - 1/6*(3*C*e*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 - 1/5*(3*C*e
^2*f + 3*B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/2)*x^2/d^2 - 1/4*(C*e^3 + 3*B*e
^2*f + 3*A*e*f^2)*(-d^2*x^2 + 1)^(3/2)*x/d^2 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A
*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1
/8*(3*C*e*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(C*e^3 + 3*B*e^2*f
+ 3*A*e*f^2)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 2/15*(3*C*e^2*f + 3*
B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/2)/d^4 + 1/16*(3*C*e*f^2 + B*f^3)*sqrt(-
d^2*x^2 + 1)*x/d^4 + 1/16*(3*C*e*f^2 + B*f^3)*arcsin(d^2*x/sqrt(d^2))/(sqrt
(d^2)*d^4)
```

Fricas [A] time = 1.10033, size = 888, normalized size = 2.14

$$\left(240\,Cd^6f^3x^6-560\,Bd^4e^3-672\,Bd^2ef^2+280\left(3\,Cd^6ef^2+Bd^6f^3\right)x^5+48\left(21\,Cd^6e^2f+21\,Bd^6ef^2+\left(7\,Ad^6-Cd^4\right)f^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1680*((240*C*d^6*f^3*x^6 - 560*B*d^4*e^3 - 672*B*d^2*e*f^2 + 280*(3*C*d^6
*e*f^2 + B*d^6*f^3)*x^5 + 48*(21*C*d^6*e^2*f + 21*B*d^6*e*f^2 + (7*A*d^6 -
C*d^4)*f^3)*x^4 - 336*(5*A*d^4 + 2*C*d^2)*e^2*f - 32*(7*A*d^2 + 4*C)*f^3 +
```

```
70*(6*C*d^6*e^3 + 18*B*d^6*e^2*f - B*d^4*f^3 + 3*(6*A*d^6 - C*d^4)*e*f^2)*x
^3 + 16*(35*B*d^6*e^3 - 21*B*d^4*e*f^2 + 21*(5*A*d^6 - C*d^4)*e^2*f - (7*A*
d^4 + 4*C*d^2)*f^3)*x^2 - 105*(6*B*d^4*e^2*f + B*d^2*f^3 - 2*(4*A*d^6 - C*d
^4)*e^3 + 3*(2*A*d^4 + C*d^2)*e*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 210*
(6*B*d^3*e^2*f + B*d*f^3 + 2*(4*A*d^5 + C*d^3)*e^3 + 3*(2*A*d^3 + C*d)*e*f^
2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.14709, size = 1122, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/1680*(112*((d*x + 1)*(3*(d*x + 1)*((d*x + 1)/d^3 - 4/d^3) + 17/d^3) - 10/
d^3)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*A*f^3 + 16*((3*((d*x + 1)*(5*(d*x + 1)*
((d*x + 1)/d^5 - 6/d^5) + 74/d^5) - 96/d^5)*(d*x + 1) + 203/d^5)*(d*x + 1)
- 70/d^5)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*C*f^3 + 336*((d*x + 1)*(3*(d*x + 1
)*((d*x + 1)/d^3 - 4/d^3) + 17/d^3) - 10/d^3)*(d*x + 1)^(3/2)*sqrt(-d*x + 1
)*B*f^2*e + 336*((d*x + 1)*(3*(d*x + 1)*((d*x + 1)/d^3 - 4/d^3) + 17/d^3) -
 10/d^3)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*C*f*e^2 + 35*(((2*((d*x + 1)*(4*(d*
x + 1)*((d*x + 1)/d^4 - 5/d^4) + 39/d^4) - 37/d^4)*(d*x + 1) + 31/d^4)*(d*x
 + 1) - 3/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x
 + 1))/d^4)*B*f^3 + 1680*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d*x + 1)*A*f*e^2/d
 + 630*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*A*f^
2*e + 105*(((2*((d*x + 1)*(4*(d*x + 1)*((d*x + 1)/d^4 - 5/d^4) + 39/d^4) -
37/d^4)*(d*x + 1) + 31/d^4)*(d*x + 1) - 3/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*C*f^2*e + 560*(d*x + 1)^(3/2)*(
d*x - 1)*sqrt(-d*x + 1)*B*e^3/d + 630*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)/d
^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*s
qrt(2)*sqrt(d*x + 1))/d^2)*B*f*e^2 + 840*(sqrt(d*x + 1)*sqrt(-d*x + 1)*d*x
+ 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e^3 + 210*(((d*x + 1)*(2*(d*x + 1)
*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2
*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C*e^3)/d
```

3.2 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=286

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(8\left(C\left(d^2e^3-4ef^2\right)-2f\left(5Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-3fx\left(5f^2\left(2Ad^2+C\right)-2d^2e(Ce-2Bf)\right)\right)}{120d^4f}+\frac{x\sqrt{1-d}}{}$$

```
[Out] ((C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*x*Sqrt[1 - d
^2*x^2])/(16*d^4) + ((C*e - 2*B*f)*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(10*d^2
*f) - (C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(6*d^2*f) + ((8*(C*(d^2*e^3 - 4*e
*f^2) - 2*f*(5*A*d^2*e*f + B*(d^2*e^2 + f^2))) - 3*f*(5*(C + 2*A*d^2)*f^2 -
 2*d^2*e*(C*e - 2*B*f))*x)*(1 - d^2*x^2)^(3/2))/(120*d^4*f) + ((C*(2*d^2*e^
2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*ArcSin[d*x])/(16*d^5)
```

Rubi [A] time = 0.563279, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 833, 780, 195, 216}

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(8\left(C\left(d^2e^3-4ef^2\right)-2f\left(5Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-3fx\left(5f^2\left(2Ad^2+C\right)-2d^2e(Ce-2Bf)\right)\right)}{120d^4f}+\frac{x\sqrt{1-d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*x*Sqrt[1 - d
^2*x^2])/(16*d^4) + ((C*e - 2*B*f)*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(10*d^2
*f) - (C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(6*d^2*f) + ((8*(C*(d^2*e^3 - 4*e
*f^2) - 2*f*(5*A*d^2*e*f + B*(d^2*e^2 + f^2))) - 3*f*(5*(C + 2*A*d^2)*f^2 -
 2*d^2*e*(C*e - 2*B*f))*x)*(1 - d^2*x^2)^(3/2))/(120*d^4*f) + ((C*(2*d^2*e^
2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*ArcSin[d*x])/(16*d^5)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)dx = \int (e+fx)^2\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$
$$= -\frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} - \frac{\int (e+fx)^2\left(-3\left(C+2Ad^2\right)f^2+3d^2f\right.}{6d^2f^2}$$
$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} + \frac{\int (}{}$$
$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} + \frac{(8}{}$$
$$= \frac{\left(C\left(2d^2e^2+f^2\right)+2d^2\left(2Bef+A\left(4d^2e^2+f^2\right)\right)\right)x\sqrt{1-d^2x^2}}{16d^4} + \frac{(C}{}$$
$$= \frac{\left(C\left(2d^2e^2+f^2\right)+2d^2\left(2Bef+A\left(4d^2e^2+f^2\right)\right)\right)x\sqrt{1-d^2x^2}}{16d^4} + \frac{(C}{}$$

Mathematica [A] time = 0.325953, size = 244, normalized size = 0.85

$$\frac{d\sqrt{1-d^2x^2}\left(10Ad^2\left(12d^2e^2x+16ef\left(d^2x^2-1\right)+3f^2x\left(2d^2x^2-1\right)\right)+4B\left(2d^4x^2\left(10e^2+15efx+6f^2x^2\right)-d^2\left(20e^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*f^
2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2) +
```

```
 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^2*x
^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d^4*x
^4))) + 15*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*Ar
cSin[d*x])/(240*d^5)
```

Maple [C] time = 0.013, size = 652, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-15*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*f^
2+40*C*csgn(d)*x^5*d^5*f^2*(-d^2*x^2+1)^(1/2)+48*B*csgn(d)*x^4*d^5*f^2*(-d^
2*x^2+1)^(1/2)+60*A*csgn(d)*x^3*d^5*f^2*(-d^2*x^2+1)^(1/2)+60*C*csgn(d)*x^3
*d^5*e^2*(-d^2*x^2+1)^(1/2)+80*B*csgn(d)*x^2*d^5*e^2*(-d^2*x^2+1)^(1/2)-10*
C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*f^2-16*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2
)*x^2*f^2-160*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e*f-30*C*csgn(d)*d^3*(-d^2*x
^2+1)^(1/2)*x*e^2-30*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*f^2+120*A*csgn(d)*d
^5*(-d^2*x^2+1)^(1/2)*x*e^2-80*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e^2+120*A*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^4*e^2+30*A*arctan(csgn(d)*d*x/(-d^2
*x^2+1)^(1/2))*d^2*f^2+30*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2+
15*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^2+60*B*arctan(csgn(d)*d*x/(-d
^2*x^2+1)^(1/2))*d^2*e*f-32*B*csgn(d)*d*(-d^2*x^2+1)^(1/2)*f^2-64*C*csgn(d)
*d*(-d^2*x^2+1)^(1/2)*e*f-60*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*e*f+96*C*cs
gn(d)*x^4*d^5*e*f*(-d^2*x^2+1)^(1/2)+120*B*csgn(d)*x^3*d^5*e*f*(-d^2*x^2+1)
^(1/2)+160*A*csgn(d)*x^2*d^5*e*f*(-d^2*x^2+1)^(1/2)-32*C*csgn(d)*d^3*(-d^2*
x^2+1)^(1/2)*x^2*e*f)*csgn(d)/(-d^2*x^2+1)^(1/2)/d^5
```

Maxima [A] time = 3.75575, size = 459, normalized size = 1.6

$$-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cf^2x^3}{6\,d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}Ae^2x+\frac{Ae^2\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Be^2}{3\,d^2}-\frac{2\left(-d^2x^2+1\right)^{\frac{3}{2}}Aef}{3\,d^2}-\frac{\left(-d^2x^2+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^2*x +
1/2*A*e^2*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^
2/d^2 - 2/3*(-d^2*x^2 + 1)^(3/2)*A*e*f/d^2 - 1/5*(-d^2*x^2 + 1)^(3/2)*(2*C*
e*f + B*f^2)*x^2/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e^2 + 2*B*e*f + A*f^2)*x
/d^2 - 1/8*(-d^2*x^2 + 1)^(3/2)*C*f^2*x/d^4 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e^2
 + 2*B*e*f + A*f^2)*x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(C*e^
2 + 2*B*e*f + A*f^2)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) + 1/16*C*f^2*a
rcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4) - 2/15*(-d^2*x^2 + 1)^(3/2)*(2*C*e*f
 + B*f^2)/d^4
```

Fricas [A] time = 1.10435, size = 617, normalized size = 2.16

$$\left(40\,Cd^5f^2x^5 - 80\,Bd^3e^2 + 48\left(2\,Cd^5ef + Bd^5f^2\right)x^4 - 32\,Bdf^2 + 10\left(6\,Cd^5e^2 + 12\,Bd^5ef + \left(6\,Ad^5 - Cd^3\right)f^2\right)x^3 - 32\,(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/240*((40*C*d^5*f^2*x^5 - 80*B*d^3*e^2 + 48*(2*C*d^5*e*f + B*d^5*f^2)*x^4
- 32*B*d*f^2 + 10*(6*C*d^5*e^2 + 12*B*d^5*e*f + (6*A*d^5 - C*d^3)*f^2)*x^3
- 32*(5*A*d^3 + 2*C*d)*e*f + 16*(5*B*d^5*e^2 - B*d^3*f^2 + 2*(5*A*d^5 - C*d
^3)*e*f)*x^2 - 15*(4*B*d^3*e*f - 2*(4*A*d^5 - C*d^3)*e^2 + (2*A*d^3 + C*d)*
f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*e*f + 2*(4*A*d^4 + C*d^2
)*e^2 + (2*A*d^2 + C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65372, size = 772, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/240*(16*((d*x + 1)*(3*(d*x + 1)*((d*x + 1)/d^3 - 4/d^3) + 17/d^3) - 10/d^
3)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*B*f^2 + 32*((d*x + 1)*(3*(d*x + 1)*((d*x
+ 1)/d^3 - 4/d^3) + 17/d^3) - 10/d^3)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*C*f*e
+ 160*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d*x + 1)*A*f*e/d + 30*(((d*x + 1)*(2*
(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*x + 1)*sqrt(-d*x
 + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*A*f^2 + 5*(((2*((d*x + 1)*
(4*(d*x + 1)*((d*x + 1)/d^4 - 5/d^4) + 39/d^4) - 37/d^4)*(d*x + 1) + 31/d^4
)*(d*x + 1) - 3/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1))/d^4)*C*f^2 + 80*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d*x + 1)*B*e^2
/d + 60*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*
sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*f
*e + 120*(sqrt(d*x + 1)*sqrt(-d*x + 1)*d*x + 2*arcsin(1/2*sqrt(2)*sqrt(d*x
+ 1)))*A*e^2 + 30*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2)
 - 1/d^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)
)/d^2)*C*e^2)/d
```

3.3 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=168

$$-\frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-C\left(3d^2e^2-2f^2\right)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}+\frac{x\sqrt{1-d^2x^2}\left(4Ad^2e+Bf+Ce\right)}{8d^2}+\frac{\sin^{-1}(d\ldots}{}$$

```
[Out] ((C*e + 4*A*d^2*e + B*f)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (C*(e + f*x)^2*(1 -
 d^2*x^2)^(3/2))/(5*d^2*f) - ((4*(5*d^2*f*(B*e + A*f) - C*(3*d^2*e^2 - 2*f^
2)) - 3*d^2*f*(3*C*e - 5*B*f)*x)*(1 - d^2*x^2)^(3/2))/(60*d^4*f) + ((C*e +
4*A*d^2*e + B*f)*ArcSin[d*x])/(8*d^3)
```

Rubi [A] time = 0.250389, antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1609, 1654, 780, 195, 216}

$$-\frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-\frac{1}{4}C\left(12d^2e^2-8f^2\right)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}+\frac{x\sqrt{1-d^2x^2}\left(4Ad^2e+Bf+Ce\right)}{8d^2}+\frac{\sin^{-}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((C*e + 4*A*d^2*e + B*f)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (C*(e + f*x)^2*(1 -
 d^2*x^2)^(3/2))/(5*d^2*f) - ((4*(5*d^2*f*(B*e + A*f) - (C*(12*d^2*e^2 - 8*
f^2))/4) - 3*d^2*f*(3*C*e - 5*B*f)*x)*(1 - d^2*x^2)^(3/2))/(60*d^4*f) + ((C
*e + 4*A*d^2*e + B*f)*ArcSin[d*x])/(8*d^3)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \int (e+fx)\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\int (e+fx)\left(-\left(2C+5Ad^2\right)f^2+d^2f(3C}{5d^2f^2}$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}C\left(12d^2e^2-8f^2\right.\right.\right.}{60d}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d\right.\right.}{}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d\right.\right.}{}$$

Mathematica [A] time = 0.170314, size = 141, normalized size = 0.84

$$\frac{\sqrt{1-d^2x^2}\left(60Ad^4ex+40Ad^2f\left(d^2x^2-1\right)+5Bd^2\left(8d^2ex^2+6d^2fx^3-8e-3fx\right)+15Cd^2ex\left(2d^2x^2-1\right)+8Cf\left(3d^4x^4\right.\right.}{120d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*x
*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) + 8*
C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 15*d*(C*e + 4*A*d^2*e + B*f)*ArcSin[d*x])
/(120*d^4)
```

Maple [C] time = 0.01, size = 377, normalized size = 2.2

$$\frac{\mathrm{csgn}(d)}{120\,d^4}\sqrt{-dx+1}\sqrt{dx+1}\left(24\,C\mathrm{csgn}(d)\,x^4d^4f\sqrt{-d^2x^2+1}+30\,B\mathrm{csgn}(d)\,x^3d^4f\sqrt{-d^2x^2+1}+30\,C\mathrm{csgn}(d)\,x^3d^4e\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*C*csgn(d)*x^4*d^4*f*(-d^2*x^2+1)^(1/
2)+30*B*csgn(d)*x^3*d^4*f*(-d^2*x^2+1)^(1/2)+30*C*csgn(d)*x^3*d^4*e*(-d^2*x
```

```
^2+1)^(1/2)+40*A*csgn(d)*x^2*d^4*f*(-d^2*x^2+1)^(1/2)+40*B*csgn(d)*x^2*d^4*
e*(-d^2*x^2+1)^(1/2)+60*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e-8*C*csgn(d)*(-
d^2*x^2+1)^(1/2)*x^2*d^2*f-15*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f-15*C*csg
n(d)*(-d^2*x^2+1)^(1/2)*x*d^2*e-40*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f+60*A*
arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e-40*B*csgn(d)*(-d^2*x^2+1)^(1/2
)*d^2*e+15*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*f-16*C*csgn(d)*(-d^2*
x^2+1)^(1/2)*f+15*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*e)*csgn(d)/d^4
/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 3.29509, size = 263, normalized size = 1.57

$$\frac{1}{2}\sqrt{-d^2x^2+1}Aex-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{Ae\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Be}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Af}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}(Ce+}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*e*x - 1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*A
*e*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 - 1
/3*(-d^2*x^2 + 1)^(3/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e + B*f)*x/d^
2 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e + B*f)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*
f/d^4 + 1/8*(C*e + B*f)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.08591, size = 386, normalized size = 2.3

$$\frac{\left(24\,Cd^4fx^4-40\,Bd^2e+30\left(Cd^4e+Bd^4f\right)x^3+8\left(5\,Bd^4e+\left(5\,Ad^4-Cd^2\right)f\right)x^2-8\left(5\,Ad^2+2\,C\right)f-15\left(Bd^2f-\left(4\,Ad^4-\right.\right.}{120\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/120*((24*C*d^4*f*x^4 - 40*B*d^2*e + 30*(C*d^4*e + B*d^4*f)*x^3 + 8*(5*B*d
^4*e + (5*A*d^4 - C*d^2)*f)*x^2 - 8*(5*A*d^2 + 2*C)*f - 15*(B*d^2*f - (4*A*
d^4 - C*d^2)*e)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3 + C*
d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.04403, size = 429, normalized size = 2.55

$$\frac{8\left((dx+1)\left(3\,(dx+1)\left(\frac{dx+1}{d^3}-\frac{4}{d^3}\right)+\frac{17}{d^3}\right)-\frac{10}{d^3}\right)(dx+1)^{\frac{3}{2}}\sqrt{-dx+1}Cf+\frac{40\,(dx+1)^{\frac{3}{2}}(dx-1)\sqrt{-dx+1}Af}{d}+\frac{40\,(dx+1)^{\frac{3}{2}}(dx-1)\sqrt{-dx+1}B}{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/120*(8*((d*x + 1)*(3*(d*x + 1)*((d*x + 1)/d^3 - 4/d^3) + 17/d^3) - 10/d^3
)*(d*x + 1)^(3/2)*sqrt(-d*x + 1)*C*f + 40*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d
*x + 1)*A*f/d + 40*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d*x + 1)*B*e/d + 15*(((d
*x + 1)*(2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*x + 1
)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*f + 60*(sqrt(
d*x + 1)*sqrt(-d*x + 1)*d*x + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e + 15
*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*
x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C*e)/d
```

3.4 $\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=95

$$\frac{x\sqrt{1-d^2x^2}\left(4Ad^2+C\right)}{8d^2}+\frac{\left(4Ad^2+C\right)\sin^{-1}(dx)}{8d^3}-\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

```
[Out] ((C + 4*A*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (B*(1 - d^2*x^2)^(3/2))/(3*d^
2) - (C*x*(1 - d^2*x^2)^(3/2))/(4*d^2) + ((C + 4*A*d^2)*ArcSin[d*x])/(8*d^3
)
```

Rubi [A] time = 0.0729961, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {899, 1815, 641, 195, 216}

$$\frac{x\sqrt{1-d^2x^2}\left(4Ad^2+C\right)}{8d^2}+\frac{\left(4Ad^2+C\right)\sin^{-1}(dx)}{8d^3}-\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((C + 4*A*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (B*(1 - d^2*x^2)^(3/2))/(3*d^
2) - (C*x*(1 - d^2*x^2)^(3/2))/(4*d^2) + ((C + 4*A*d^2)*ArcSin[d*x])/(8*d^3
)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)dx = \int \left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$
$$= -\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} - \frac{\int \left(-C-4Ad^2-4Bd^2x\right)\sqrt{1-d^2x^2}\,dx}{4d^2}$$
$$= -\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} - \frac{\left(-C-4Ad^2\right)\int \sqrt{1-d^2x^2}\,dx}{4d^2}$$
$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} + \frac{\left(C+4Ad^2\right)}{8}$$
$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} + \frac{\left(C+4Ad^2\right)}{8d}$$

Mathematica [A] time = 0.0622698, size = 71, normalized size = 0.75

$$\frac{d\sqrt{1-d^2x^2}\left(12Ad^2x+8Bd^2x^2-8B+6Cd^2x^3-3Cx\right)+3\left(4Ad^2+C\right)\sin^{-1}(dx)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^3
) + 3*(C + 4*A*d^2)*ArcSin[d*x])/(24*d^3)
```

Maple [C] time = 0.011, size = 185, normalized size = 2.

$$\frac{\mathrm{csgn}(d)}{24\,d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(6\,C\mathrm{csgn}(d)\,x^3d^3\sqrt{-d^2x^2+1}+8\,B\mathrm{csgn}(d)\,x^2d^3\sqrt{-d^2x^2+1}+12\,A\mathrm{csgn}(d)\,d^3\sqrt{-d^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*C*csgn(d)*x^3*d^3*(-d^2*x^2+1)^(1/2)+8
*B*csgn(d)*x^2*d^3*(-d^2*x^2+1)^(1/2)+12*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x
-3*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x+12*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1
/2))*d^2-8*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d+3*C*arctan(csgn(d)*d*x/(-d^2*x^2+
1)^(1/2)))*csgn(d)/(-d^2*x^2+1)^(1/2)/d^3
```

Maxima [A] time = 4.04605, size = 154, normalized size = 1.62

$$\frac{1}{2}\sqrt{-d^2x^2+1}Ax - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cx}{4\,d^2} + \frac{A\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}} - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}B}{3\,d^2} + \frac{\sqrt{-d^2x^2+1}Cx}{8\,d^2} + \frac{C\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{8\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcsi
n(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d
^2*x^2 + 1)*C*x/d^2 + 1/8*C*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.04244, size = 224, normalized size = 2.36

$$\frac{\left(6\,Cd^{3}x^{3}+8\,Bd^{3}x^{2}-8\,Bd+3\left(4\,Ad^{3}-Cd\right)x\right)\sqrt{dx+1}\sqrt{-dx+1}-6\left(4\,Ad^{2}+C\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{24\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x +
1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.89606, size = 198, normalized size = 2.08

$$\frac{\frac{8\,(dx+1)^{\frac{3}{2}}(dx-1)\sqrt{-dx+1}B}{d}+12\left(\sqrt{dx+1}\sqrt{-dx+1}dx+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)\right)A+3\left(\left((dx+1)\left(2\,(dx+1)\left(\frac{dx+1}{d^{2}}-\frac{3}{d^{2}}\right)+\frac{5}{d^{2}}\right.\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(8*(d*x + 1)^(3/2)*(d*x - 1)*sqrt(-d*x + 1)*B/d + 12*(sqrt(d*x + 1)*sq
rt(-d*x + 1)*d*x + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A + 3*(((d*x + 1)*(
2*(d*x + 1)*((d*x + 1)/d^2 - 3/d^2) + 5/d^2) - 1/d^2)*sqrt(d*x + 1)*sqrt(-d
*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C)/d
```

3.5 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal. Leaf size=122

$$\frac{(Af^2-Bef+Ce^2)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rubi [A] time = 0.310779, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 844, 216, 725, 204}

$$\frac{(Af^2-Bef+Ce^2)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$
$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(Ce^2-Bef+Af^2\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{f^2}$$
$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{\left(Ce^2-Bef+Af^2\right)\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \right.}{f^2}$$
$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{\left(Ce^2-Bef+Af^2\right)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

Mathematica [A] time = 0.145378, size = 117, normalized size = 0.96

$$\frac{\frac{\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{\sqrt{d^2e^2-f^2}} + \frac{\sin^{-1}(dx)(Bf-Ce)}{d} - \frac{Cf\sqrt{1-d^2x^2}}{d^2}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + ((-(C*e) + B*f)*ArcSin[d*x])/d + ((C*e^2
+ f*(-(B*e) + A*f))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*
x^2])])/Sqrt[d^2*e^2 - f^2])/f^2
```

Maple [C] time = 0.04, size = 373, normalized size = 3.1

$$\frac{\text{csgn}(d)}{f^3d^2}\left(-A\text{csgn}(d)\ln\left(2\frac{1}{fx+e}\left(d^2ex+\sqrt{-\frac{d^2e^2-f^2}{f^2}}\sqrt{-d^2x^2+1}f+f\right)\right)d^2f^2+B\text{csgn}(d)\ln\left(2\frac{1}{fx+e}\left(d^2ex+\sqrt{-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f
)/(f*x+e))*d^2*f^2+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2
*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e*f-C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)
/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^2+B*arctan(csgn(d)*d*x/(
-d^2*x^2+1)^(1/2))*d*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*csgn(d)*f^2*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f*(-(d^2*e^2-f^2)/f^2)^(1/2))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)/(
-(d^2*e^2-f^2)/f^2)^(1/2)/f^3/(-d^2*x^2+1)^(1/2)/d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 29.8818, size = 1019, normalized size = 8.35

$$\left[-\frac{\left(Cd^2e^2 - Bd^2ef + Ad^2f^2\right)\sqrt{-d^2e^2 + f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}\left(d^2ex+f\right)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+\left(d^2e^2-f^2\right)\sqrt{-dx+1}\right)\sqrt{dx+1}}{fx+e}\right) + (C}{d^4e^2f^2 - d^2f^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\left(e + fx\right)\sqrt{-dx + 1}\sqrt{dx + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.6 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rubi [A] time = 0.330582, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1651, 844, 216, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```


```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```


Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\int \frac{}{(e+fx}}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\text{Subst}\left(\int\right.}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\tan^{-1}\left(\frac{}{\sqrt{d}}\right.}{\left(d^2e^2-f^2\right)^{3/2}}$$

Mathematica [A] time = 0.442404, size = 211, normalized size = 1.29

$$\frac{-\frac{f\sqrt{1-d^2x^2}\left(f(Af-Be)+Ce^2\right)}{\left(f^2-d^2e^2\right)(e+fx)} - \frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{\log(e+fx)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{C\sin^{-1}(d}{d}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

Maple [C] time = 0.041, size = 899, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f
)/(f*x+e))*x*d^3*e*f^3+C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(
-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^3*e^3*f-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*
e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x*d^2*e^2*f^2*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)
/(f*x+e))*x*d*f^4-B*csgn(d)*d*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1
/2)*f+f)/(f*x+e))*x*d*e*f^3+C*csgn(d)*d*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*
(-d^2*x^2+1)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-
d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2
-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*x*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(d*x+1)^
(1/2)*(-d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(f*x+e)/d/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

<u>Maxima</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

<u>Fricas</u> [B] time = 118.982, size = 2082, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
```

```
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.7 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{}$$

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rubi [A] time = 0.355188, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1651, 807, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} + \frac{\int \frac{2\left(Ce+Ad^2e-Bf\right)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2\left(d^2e^2-f^2\right)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1-}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1-}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1-}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

Mathematica [A] time = 0.395583, size = 273, normalized size = 1.1

$$\frac{1}{2}\left(-\frac{\sqrt{1-d^2x^2}\left(-Ad^2ef(4e+3fx)+Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)+Ce\left(d^2e^2x-3ef-4f^2x\right)\right)}{\left(f^2-d^2e^2\right)^2(e+fx)^2} - \frac{\log\left(\sqrt{1-d^2}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

Maple [C] time = 0.045, size = 1449, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f
*x+e))*x^2*d^2*f^4-3*A*x*d^2*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^
(1/2)+B*x*d^2*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+C*x*d^2
*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)-6*B*ln(2*(d^2*e*x+(-(d
^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^2*e^2*f^2+2*C*l
n(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*
d^2*e^3*f-4*A*d^2*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*B
*d^2*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+B*e*f^3*(-(d^2*e^2
-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1
/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*ln(2*(d^2*e*x+(-(d^2*e
^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2
*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*e*f^3-3*
C*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*B*x*f^4*(-(d^2*e^
2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)-4*C*x*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)
*(-d^2*x^2+1)^(1/2)+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+
1)^(1/2)*f+f)/(f*x+e))*x^2*d^4*e^2*f^2+4*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^
2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^4*e^3*f-3*B*ln(2*(d^2*e*x+(-(
d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x^2*d^2*e*f^3+C*ln
(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x^2
*d^2*e^2*f^2+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2
)*f+f)/(f*x+e))*x*d^2*e*f^3+A*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(
1/2)+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(
f*x+e))*d^4*e^4+2*C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(
1/2)*f+f)/(f*x+e))*x^2*f^4+C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2
*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^
(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*e^2*f^2)*csgn(d)^2*(d*x+1)^(1/2)*(-d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44776, size = 3105, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
```

```
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.8 $\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}} dx$

Optimal. Leaf size=340

$$\frac{\sqrt{1-d^2x^2}\left(d^2fx\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2\right)+4\left(C\left(-52d^2e^2f^2+3d^4e^4-16f^4\right)-5d^2f\left(\right.\right.}{120d^6f}$$

```
[Out] -((4*(4*C + 5*A*d^2)*f^2 - 3*d^2*e*(C*e - 5*B*f))*(e + f*x)^2*Sqrt[1 - d^2*
x^2])/(60*d^4*f) + ((C*e - 5*B*f)*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(20*d^2*f)
 - (C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(5*d^2*f) + ((4*(C*(3*d^4*e^4 - 52*d^2
*e^2*f^2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4*e*
f^2))) + d^2*f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 - 100*A*d^2*e*f^2
 - 45*B*f^3)*x)*Sqrt[1 - d^2*x^2])/(120*d^6*f) + ((4*C*d^2*e^3 + 8*A*d^4*e^
3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSin[d*x])/(8*
d^5)
```

Rubi [A] time = 0.632967, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1654, 833, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}(e+fx)^2\left(5f(4Af+3Be)-C\left(3e^2-\frac{16f^2}{d^2}\right)\right)}{60d^2f}+\frac{\sqrt{1-d^2x^2}\left(d^2fx\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Ca\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2)/d^2))*(e + f*x)^2*Sqrt[1 - d^2
*x^2])/(60*d^2*f) + ((C*e - 5*B*f)*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(20*d^2*f
) - (C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(5*d^2*f) + ((4*(C*(3*d^4*e^4 - 52*d^
2*e^2*f^2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4*e
*f^2))) + d^2*f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 - 100*A*d^2*e*f^
2 - 45*B*f^3)*x)*Sqrt[1 - d^2*x^2])/(120*d^6*f) + ((4*C*d^2*e^3 + 8*A*d^4*e
^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSin[d*x])/(8
*d^5)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} - \frac{\int \frac{(e+fx)^3\left(-\left(4C+5Ad^2\right)f^2+d^2f(Ce-5Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{5d^2f^2}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} + \frac{\int \frac{(e+fx)^2\left(d^2f^2\left(13Ce+20Ad^2e+15B\right.\right.}{}}{}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1}}{20d^2f}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1}}{20d^2f}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1}}{20d^2f}$$

Mathematica [A] time = 0.37035, size = 241, normalized size = 0.71

$$\frac{15d\sin^{-1}(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)-\sqrt{1-d^2x^2}\left(20Ad^2f\left(d^2\left(18e^2+9efx+2f^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^2
)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2
```

```
+ f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d^4*
x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 15*d*(4*C*d^2*e^3 +
 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSi
n[d*x])/(120*d^6)
```

Maple [C] time = 0.025, size = 643, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^4*d^
4*f^3+30*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x^3*d^4*f^3+90*C*csgn(d)*(-d^2*x^2+1)
^(1/2)*x^3*d^4*e*f^2+40*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*f^3+120*B*csgn
(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*e*f^2+120*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d
^4*e^2*f+180*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e*f^2+180*B*csgn(d)*(-d^2*x
^2+1)^(1/2)*x*d^4*e^2*f+60*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e^3+360*A*csg
n(d)*(-d^2*x^2+1)^(1/2)*d^4*e^2*f-120*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/
2))*d^5*e^3+120*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^4*e^3+32*C*csgn(d)*(-d^2*x^2
+1)^(1/2)*x^2*d^2*f^3+45*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f^3+135*C*csgn(
d)*(-d^2*x^2+1)^(1/2)*x*d^2*e*f^2+80*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f^3-1
80*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e*f^2+240*B*csgn(d)*(-d^2*x
^2+1)^(1/2)*d^2*e*f^2-180*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^3*e^2*
f+240*C*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*e^2*f-60*C*arctan(csgn(d)*d*x/(-d^2*
x^2+1)^(1/2))*d^3*e^3-45*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d*f^3+64*
C*csgn(d)*(-d^2*x^2+1)^(1/2)*f^3-135*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f^2)*csgn(d)/d^6/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 3.19919, size = 524, normalized size = 1.54

$$-\frac{\sqrt{-d^2x^2+1}Cf^3x^4}{5\,d^2}+\frac{Ae^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}Be^3}{d^2}-\frac{3\sqrt{-d^2x^2+1}Ae^2f}{d^2}-\frac{4\sqrt{-d^2x^2+1}Cf^3x^2}{15\,d^4}-\frac{\left(3\,Cef^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 + A*e^3*arcsin(d^2*x/sqrt(d^2))/sqrt(
d^2) - sqrt(-d^2*x^2 + 1)*B*e^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*A*e^2*f/d^2 - 4/
15*sqrt(-d^2*x^2 + 1)*C*f^3*x^2/d^4 - 1/4*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2
 + 1)*x^3/d^2 - 1/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)*x^2/
d^2 - 1/2*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 + 1/2*(C
*e^3 + 3*B*e^2*f + 3*A*e*f^2)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 8/1
5*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)
*x/d^4 - 2/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(
3*C*e*f^2 + B*f^3)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4)
```

Fricas [A] time = 1.14323, size = 644, normalized size = 1.89

$$-\frac{\left(24\, Cd^4 f^3 x^4 + 120\, Bd^4 e^3 + 240\, Bd^2 e f^2 + 120\left(3\, Ad^4 + 2\, Cd^2\right)e^2 f + 16\left(5\, Ad^2 + 4\, C\right)f^3 + 30\left(3\, Cd^4 e f^2 + Bd^4 f^3\right)x^3 + 8 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/120*((24*C*d^4*f^3*x^4 + 120*B*d^4*e^3 + 240*B*d^2*e*f^2 + 120*(3*A*d^4
+ 2*C*d^2)*e^2*f + 16*(5*A*d^2 + 4*C)*f^3 + 30*(3*C*d^4*e*f^2 + B*d^4*f^3)*
x^3 + 8*(15*C*d^4*e^2*f + 15*B*d^4*e*f^2 + (5*A*d^4 + 4*C*d^2)*f^3)*x^2 + 1
5*(4*C*d^4*e^3 + 12*B*d^4*e^2*f + 3*B*d^2*f^3 + 3*(4*A*d^4 + 3*C*d^2)*e*f^2
)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*e^2*f + 3*B*d*f^3 + 4*(2*A
*d^5 + C*d^3)*e^3 + 3*(4*A*d^3 + 3*C*d)*e*f^2)*arctan((sqrt(d*x + 1)*sqrt(-
d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.27081, size = 551, normalized size = 1.62

$$-\frac{\left(360\, Ad^{29} f e^2 - 180\, Ad^{28} f^2 e + 120\, Ad^{27} f^3 + 120\, Bd^{29} e^3 - 180\, Bd^{28} f e^2 + 360\, Bd^{27} f^2 e - 75\, Bd^{26} f^3 - 60\, Cd^{28} e^3 + 360\, C \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2211840*((360*A*d^29*f*e^2 - 180*A*d^28*f^2*e + 120*A*d^27*f^3 + 120*B*d
^29*e^3 - 180*B*d^28*f*e^2 + 360*B*d^27*f^2*e - 75*B*d^26*f^3 - 60*C*d^28*e
^3 + 360*C*d^27*f*e^2 - 225*C*d^26*f^2*e + 120*C*d^25*f^3 + (180*A*d^28*f^2
*e - 80*A*d^27*f^3 + 180*B*d^28*f*e^2 - 240*B*d^27*f^2*e + 135*B*d^26*f^3 +
 60*C*d^28*e^3 - 240*C*d^27*f*e^2 + 405*C*d^26*f^2*e - 160*C*d^25*f^3 + 2*(
20*A*d^27*f^3 + 60*B*d^27*f^2*e - 45*B*d^26*f^3 + 60*C*d^27*f*e^2 - 135*C*d
^26*f^2*e + 88*C*d^25*f^3 + 3*(4*(d*x + 1)*C*d^25*f^3 + 5*B*d^26*f^3 + 15*C
*d^26*f^2*e - 16*C*d^25*f^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)
*sqrt(-d*x + 1) - 30*(8*A*d^30*e^3 + 12*A*d^28*f^2*e + 12*B*d^28*f*e^2 + 3*
B*d^26*f^3 + 4*C*d^28*e^3 + 9*C*d^26*f^2*e)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))/d
```

3.9 $\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{\sqrt{1-d^2x^2}\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-fx\left(3f^2\left(4Ad^2+3C\right)-2d^2e(Ce-4Bf)\right)\right)}{24d^4f}+\sin^{-1}(d$$

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(12*d^2*f) - (C*(e + f*x)^3*S
qrt[1 - d^2*x^2])/(4*d^2*f) + ((4*(C*(d^2*e^3 - 8*e*f^2) - 4*f*(3*A*d^2*e*f
 + B*(d^2*e^2 + f^2))) - f*(3*(3*C + 4*A*d^2)*f^2 - 2*d^2*e*(C*e - 4*B*f))*
x)*Sqrt[1 - d^2*x^2])/(24*d^4*f) + ((C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f
 + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(8*d^5)
```

Rubi [A] time = 0.492606, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1654, 833, 780, 216}

$$\frac{\sqrt{1-d^2x^2}\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-fx\left(3f^2\left(4Ad^2+3C\right)-2d^2e(Ce-4Bf)\right)\right)}{24d^4f}+\sin^{-1}(d$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```


```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(12*d^2*f) - (C*(e + f*x)^3*S
qrt[1 - d^2*x^2])/(4*d^2*f) + ((4*(C*(d^2*e^3 - 8*e*f^2) - 4*f*(3*A*d^2*e*f
 + B*(d^2*e^2 + f^2))) - f*(3*(3*C + 4*A*d^2)*f^2 - 2*d^2*e*(C*e - 4*B*f))*
x)*Sqrt[1 - d^2*x^2])/(24*d^4*f) + ((C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f
 + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(8*d^5)
```


Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} - \frac{\int \frac{(e+fx)^2\left(-\left(3C+4Ad^2\right)f^2+d^2f(Ce-4Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{4d^2f^2}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\int \frac{(e+fx)\left(d^2f^2\left(7Ce+12Ad^2e+8Bf\right)\cdots\right.}{\sqrt{\cdots}}}{1\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3\cdots\right.\right.\right.}{\cdots}$$

Mathematica [A] time = 0.207138, size = 160, normalized size = 0.7

$$\frac{3\sin^{-1}(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)-d\sqrt{1-d^2x^2}\left(12Ad^2f(4e+fx)+8B\left(d^2\left(3e^2+3efx+f^2\cdots\right.\right.\right.}{24d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f*(
2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f*x
 + f^2*x^2)))) + 3*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^2 +
 f^2)))*ArcSin[d*x])/(24*d^5)
```

Maple [C] time = 0.024, size = 423, normalized size = 1.9

$$-\frac{\mathrm{csgn}(d)}{24\,d^5}\sqrt{-dx+1}\sqrt{dx+1}\left(6\,C\mathrm{csgn}(d)\,d^3\sqrt{-d^2x^2+1}x^3f^2+8\,B\mathrm{csgn}(d)\,d^3\sqrt{-d^2x^2+1}x^2f^2+16\,C\mathrm{csgn}(d)\,d^3\sqrt{-d^2x^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*
f^2+8*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^2*f^2+16*C*csgn(d)*d^3*(-d^2*x^2+1
)^(1/2)*x^2*e*f+12*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*f^2+24*B*csgn(d)*d^3*
(-d^2*x^2+1)^(1/2)*x*e*f+12*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*e^2+48*A*csg
n(d)*d^3*(-d^2*x^2+1)^(1/2)*e*f-24*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))
*d^4*e^2+24*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e^2+9*C*csgn(d)*d*(-d^2*x^2+1)
^(1/2)*x*f^2-12*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*f^2+16*B*csgn(
d)*d*(-d^2*x^2+1)^(1/2)*f^2-24*B*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2
*e*f+32*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*e*f-12*C*arctan(csgn(d)*d*x/(-d^2*x^
2+1)^(1/2))*d^2*e^2-9*C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^2)*csgn(d)
/d^5/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 4.34213, size = 356, normalized size = 1.56

$$-\frac{\sqrt{-d^2x^2+1}Cf^2x^3}{4\,d^2}+\frac{Ae^2\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}Be^2}{d^2}-\frac{2\,\sqrt{-d^2x^2+1}Aef}{d^2}-\frac{\sqrt{-d^2x^2+1}\left(2\,Cef+Bf^2\right)x^2}{3\,d^2}-\frac{\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*e^2*arcsin(d^2*x/sqrt(d^2))/sqrt(
d^2) - sqrt(-d^2*x^2 + 1)*B*e^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*e*f/d^2 - 1/3*
sqrt(-d^2*x^2 + 1)*(2*C*e*f + B*f^2)*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e^
2 + 2*B*e*f + A*f^2)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/2*(C*e^
2 + 2*B*e*f + A*f^2)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) + 3/8*C*f^2*ar
csin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4) - 2/3*sqrt(-d^2*x^2 + 1)*(2*C*e*f + B
*f^2)/d^4
```

Fricas [A] time = 1.14281, size = 435, normalized size = 1.91

$$-\frac{\left(6\,Cd^3f^2x^3+24\,Bd^3e^2+16\,Bdf^2+16\left(3\,Ad^3+2\,Cd\right)ef+8\left(2\,Cd^3ef+Bd^3f^2\right)x^2+3\left(4\,Cd^3e^2+8\,Bd^3ef+\left(4\,Ad^3\right.\right.\right.}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/24*((6*C*d^3*f^2*x^3 + 24*B*d^3*e^2 + 16*B*d*f^2 + 16*(3*A*d^3 + 2*C*d)*
e*f + 8*(2*C*d^3*e*f + B*d^3*f^2)*x^2 + 3*(4*C*d^3*e^2 + 8*B*d^3*e*f + (4*A
*d^3 + 3*C*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*e*f + 4*(2*
A*d^4 + C*d^2)*e^2 + (4*A*d^2 + 3*C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x +
 1) - 1)/(d*x)))/d^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.82415, size = 352, normalized size = 1.54

$$-\frac{\left(48\,Ad^{19}fe-12\,Ad^{18}f^{2}+24\,Bd^{19}e^{2}-24\,Bd^{18}fe+24\,Bd^{17}f^{2}-12\,Cd^{18}e^{2}+48\,Cd^{17}fe-15\,Cd^{16}f^{2}+\left(12\,Ad^{18}f^{2}+24\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/86016*((48*A*d^19*f*e - 12*A*d^18*f^2 + 24*B*d^19*e^2 - 24*B*d^18*f*e +
24*B*d^17*f^2 - 12*C*d^18*e^2 + 48*C*d^17*f*e - 15*C*d^16*f^2 + (12*A*d^18*
f^2 + 24*B*d^18*f*e - 16*B*d^17*f^2 + 12*C*d^18*e^2 - 32*C*d^17*f*e + 27*C*
d^16*f^2 + 2*(3*(d*x + 1)*C*d^16*f^2 + 4*B*d^17*f^2 + 8*C*d^17*f*e - 9*C*d^
16*f^2)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*A*d^20*e^
2 + 4*A*d^18*f^2 + 8*B*d^18*f*e + 4*C*d^18*e^2 + 3*C*d^16*f^2)*arcsin(1/2*s
qrt(2)*sqrt(d*x + 1)))/d
```

3.10 $\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3d^2f(Af+Be)-C\left(d^2e^2-2f^2\right)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f}+\frac{\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)}{2d^3}-\frac{C\sqrt{1-d^2x^2}(}{3d^2f}$$

```
[Out] -(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(3*d^2*f) - ((2*(3*d^2*f*(B*e + A*f) - C
*(d^2*e^2 - 2*f^2)) - d^2*f*(C*e - 3*B*f)*x)*Sqrt[1 - d^2*x^2])/(6*d^4*f) +
 ((C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.229777, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1609, 1654, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3d^2f(Af+Be)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f}+\frac{\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)}{2d^3}-\frac{C\sqrt{1-d^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(3*d^2*f) - ((2*(3*d^2*f*(B*e + A*f) - (
C*(2*d^2*e^2 - 4*f^2))/2) - d^2*f*(C*e - 3*B*f)*x)*Sqrt[1 - d^2*x^2])/(6*d^
4*f) + ((C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d^3)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$=-\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f}-\frac{\int \frac{(e+fx)\left(-\left(2C+3Ad^2\right)f^2+d^2f(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2f^2}$$

$$=-\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f}-\frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2f(Ce-3Bf)x\right)}{6d^4f}$$

$$=-\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f}-\frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2f(Ce-3Bf)x\right)}{6d^4f}$$

Mathematica [A] time = 0.100861, size = 88, normalized size = 0.68

$$\frac{3d\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)-\sqrt{1-d^2x^2}\left(6Ad^2f+3Bd^2(2e+fx)+C\left(3d^2ex+2d^2fx^2+4f\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(6*A*d^2*f + 3*B*d^2*(2*e + f*x) + C*(4*f + 3*d^2*e*x
+ 2*d^2*f*x^2))) + 3*d*(C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(6*d^4)
```

Maple [C] time = 0.018, size = 235, normalized size = 1.8

$$-\frac{\text{csgn}(d)}{6\,d^4}\sqrt{-dx+1}\sqrt{dx+1}\left(2\,C\text{csgn}(d)\sqrt{-d^2x^2+1}x^2d^2f+3\,B\text{csgn}(d)\sqrt{-d^2x^2+1}xd^2f+3\,C\text{csgn}(d)\sqrt{-d^2x^2+1}xd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^2*f
+3*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f+3*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^
2*e+6*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f-6*A*arctan(csgn(d)*d*x/(-d^2*x^2+1
)^(1/2))*d^3*e+6*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*e-3*B*arctan(csgn(d)*d*x/
(-d^2*x^2+1)^(1/2))*d*f+4*C*csgn(d)*(-d^2*x^2+1)^(1/2)*f-3*C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*d*e)*csgn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 3.60375, size = 205, normalized size = 1.58

$$-\frac{\sqrt{-d^2x^2+1}Cfx^2}{3\,d^2}+\frac{Ae\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}Be}{d^2}-\frac{\sqrt{-d^2x^2+1}Af}{d^2}-\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{2\,d^2}+\frac{(Ce+Bf)\arcsin}{2\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*e*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2)
 - sqrt(-d^2*x^2 + 1)*B*e/d^2 - sqrt(-d^2*x^2 + 1)*A*f/d^2 - 1/2*sqrt(-d^2*
x^2 + 1)*(C*e + B*f)*x/d^2 + 1/2*(C*e + B*f)*arcsin(d^2*x/sqrt(d^2))/(sqrt(
d^2)*d^2) - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^4
```

Fricas [A] time = 1.08703, size = 267, normalized size = 2.05

$$-\frac{\left(2\,Cd^2fx^2+6\,Bd^2e+2\left(3\,Ad^2+2\,C\right)f+3\left(Cd^2e+Bd^2f\right)x\right)\sqrt{dx+1}\sqrt{-dx+1}+6\left(Bdf+\left(2\,Ad^3+Cd\right)e\right)\arctan\left(\right.}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*((2*C*d^2*f*x^2 + 6*B*d^2*e + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*e + B*d^2
*f)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arctan(
(sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [C] time = 112.876, size = 617, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*A*e*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + A*e*meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)*d) - I*A*f*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4,
 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - A*f*meijerg(((-1
, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_po
lar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*B*e*meijerg(((-1/4, 1/4),
(0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**
(3/2)*d**2) - B*e*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4)
, (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
- I*B*f*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4,
 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + B*f*meijerg(((-3/2, -5/4,
-1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(-2*I*
pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*C*e*meijerg(((-3/4, -1/4), (-1/2, -
1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2
)*d**3) + C*e*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4),
 (-3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I
*C*f*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/
2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**4) - C*f*meijerg(((-2, -7/4, -3/
2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**4)
```

Giac [A] time = 1.98318, size = 186, normalized size = 1.43

$$-\frac{\left(6\,Ad^{11}f+6\,Bd^{11}e-3\,Bd^{10}f-3\,Cd^{10}e+6\,Cd^{9}f+\left(2\,(dx+1)Cd^{9}f+3\,Bd^{10}f+3\,Cd^{10}e-4\,Cd^{9}f\right)(dx+1)\right)\sqrt{dx+1}\sqrt{}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] -1/3840*((6*A*d^11*f + 6*B*d^11*e - 3*B*d^10*f - 3*C*d^10*e + 6*C*d^9*f + (
2*(d*x + 1)*C*d^9*f + 3*B*d^10*f + 3*C*d^10*e - 4*C*d^9*f)*(d*x + 1))*sqrt(
d*x + 1)*sqrt(-d*x + 1) - 6*(2*A*d^12*e + B*d^10*f + C*d^10*e)*arcsin(1/2*s
qrt(2)*sqrt(d*x + 1)))/d
```

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2Ad^2+C)\sin^{-1}(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] -((B*Sqrt[1 - d^2*x^2])/d^2) - (C*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.0608195, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2Ad^2+C)\sin^{-1}(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((B*Sqrt[1 - d^2*x^2])/d^2) - (C*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-C-2Ad^2-2Bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-C-2Ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(C+2Ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.0340787, size = 45, normalized size = 0.71

$$\frac{\left(2Ad^2+C\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}(2B+Cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*B + C*x)*Sqrt[1 - d^2*x^2]) + (C + 2*A*d^2)*ArcSin[d*x])/(2*d^3)
```

Maple [C] time = 0.016, size = 117, normalized size = 1.9

$$\frac{\mathrm{csgn}(d)}{2\,d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(2\,A\arctan\left(\frac{\mathrm{csgn}(d)\,dx}{\sqrt{-d^2x^2+1}}\right)d^2 - C\mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}x - 2\,B\sqrt{-d^2x^2+1}\mathrm{csgn}(d)\,d + C\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(
1/2))*d^2-C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x-2*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d
+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A] time = 3.77625, size = 105, normalized size = 1.67

$$\frac{A\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{\sqrt{-d^2x^2+1}Cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}B}{d^2} + \frac{C\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\,\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt
(-d^2*x^2 + 1)*B/d^2 + 1/2*C*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.01916, size = 167, normalized size = 2.65

$$-\frac{(Cdx+2\,Bd)\sqrt{dx+1}\sqrt{-dx+1}+2\left(2\,Ad^2+C\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [C] time = 20.572, size = 282, normalized size = 4.48

$$-\frac{iAG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{AG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iBG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0\ldots\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \ldots\end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*A*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + A*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*B*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - B*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*C*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + C*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 3.10339, size = 97, normalized size = 1.54

$$-\frac{\left((dx+1)Cd^4+2\,Bd^5-Cd^4\right)\sqrt{dx+1}\sqrt{-dx+1}-2\left(2\,Ad^6+Cd^4\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(((d*x + 1)*C*d^4 + 2*B*d^5 - C*d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
2*(2*A*d^6 + C*d^4)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.12 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal. Leaf size=122

$$\frac{\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rubi [A] time = 0.282661, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 844, 216, 725, 204}

$$\frac{\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx &= \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2} \\
&= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(Ce^2-Bef+Af^2\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{f^2} \\
&= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{\left(Ce^2-Bef+Af^2\right)\operatorname{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx,\right.}{f^2} \\
&= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{\left(Ce^2-Bef+Af^2\right)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}
\end{aligned}$$

Mathematica [A] time = 0.127119, size = 117, normalized size = 0.96

$$\frac{\frac{\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{\sqrt{d^2e^2-f^2}}+\frac{\sin^{-1}(dx)(Bf-Ce)}{d}-\frac{Cf\sqrt{1-d^2x^2}}{d^2}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + ((-(C*e) + B*f)*ArcSin[d*x])/d + ((C*e^2
+ f*(-(B*e) + A*f))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*
x^2])])/Sqrt[d^2*e^2 - f^2])/f^2
```

Maple [C] time = 0., size = 373, normalized size = 3.1

$$\frac{\operatorname{csgn}(d)}{f^3d^2}\left(-A\operatorname{csgn}(d)\ln\left(2\frac{1}{fx+e}\left(d^2ex+\sqrt{-\frac{d^2e^2-f^2}{f^2}}\sqrt{-d^2x^2+1}f+f\right)\right)d^2f^2+B\operatorname{csgn}(d)\ln\left(2\frac{1}{fx+e}\left(d^2ex+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f
)/(f*x+e))*d^2*f^2+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2
*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e*f-C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)
/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^2+B*arctan(csgn(d)*d*x/(
-d^2*x^2+1)^(1/2))*d*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*csgn(d)*f^2*(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*d*e*f*(-(d^2*e^2-f^2)/f^2)^(1/2))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)/(
-(d^2*e^2-f^2)/f^2)^(1/2)/f^3/(-d^2*x^2+1)^(1/2)/d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 30.0132, size = 1019, normalized size = 8.35

$$\left[-\frac{\left(Cd^2e^2 - Bd^2ef + Ad^2f^2\right)\sqrt{-d^2e^2 + f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}\left(d^2ex+f\right)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+\left(d^2e^2-f^2\right)\sqrt{-dx+1}\right)\sqrt{dx+1}}{fx+e}\right) + \left(Cd^2e\right.}{d^4e^2f^2 - d^2f^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\left(e + fx\right)\sqrt{-dx + 1}\sqrt{dx + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rubi [A] time = 0.295465, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1651, 844, 216, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```


```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```


Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\int \frac{\quad}{(e}}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\text{Subst}}{d^2e^2-f}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\tan^{-1}\Big(}{\left(d^2e^2-f^2\right)^{3/2}}$$

Mathematica [A] time = 0.412729, size = 211, normalized size = 1.29

$$\frac{-\frac{f\sqrt{1-d^2x^2}\left(f(Af-Be)+Ce^2\right)}{\left(f^2-d^2e^2\right)(e+fx)} - \frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{\log(e+fx)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{C\sin}{}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

Maple [C] time = 0., size = 899, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f
)/(f*x+e))*x*d^3*e*f^3+C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(
-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^3*e^3*f-A*csgn(d)*ln(2*(d^2*e*x+(-(d^2*
e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x*d^2*e^2*f^2*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)
/(f*x+e))*x*d*f^4-B*csgn(d)*d*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1
/2)*f+f)/(f*x+e))*x*d*e*f^3+C*csgn(d)*d*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*
(-d^2*x^2+1)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-
d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-(d^2*e^2
-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*x*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(d*x+1)^
(1/2)*(-d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(f*x+e)/d/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 117.112, size = 2082, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
```

```
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.14 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(\text{C}$$

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rubi [A] time = 0.328948, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1651, 807, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(\text{C}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```


```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```


Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} + \frac{\int \frac{2\left(Ce+Ad^2e-Bf\right)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2\left(d^2e^2-f^2\right)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3\right)\sqrt{1}}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

Mathematica [A] time = 0.178906, size = 273, normalized size = 1.1

$$\frac{1}{2}\left(-\frac{\sqrt{1-d^2x^2}\left(-Ad^2ef(4e+3fx)+Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)+Ce\left(d^2e^2x-3ef-4f^2x\right)\right)}{\left(f^2-d^2e^2\right)^2(e+fx)^2} - \frac{\log\left(\sqrt{1-\right.}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

Maple [C] time = 0., size = 1449, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f
*x+e))*x^2*d^2*f^4-3*A*x*d^2*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^
(1/2)+B*x*d^2*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+C*x*d^2
*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)-6*B*ln(2*(d^2*e*x+(-(d
^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^2*e^2*f^2+2*C*l
n(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*
d^2*e^3*f-4*A*d^2*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*B
*d^2*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+B*e*f^3*(-(d^2*e^2
-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1
/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*ln(2*(d^2*e*x+(-(d^2*e
^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2
*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*e*f^3-3*
C*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*B*x*f^4*(-(d^2*e^
2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)-4*C*x*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)
*(-d^2*x^2+1)^(1/2)+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+
1)^(1/2)*f+f)/(f*x+e))*x^2*d^4*e^2*f^2+4*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^
2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x*d^4*e^3*f-3*B*ln(2*(d^2*e*x+(-(
d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x^2*d^2*e*f^3+C*ln
(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*x^2
*d^2*e^2*f^2+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2
)*f+f)/(f*x+e))*x*d^2*e*f^3+A*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(
1/2)+2*A*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(
f*x+e))*d^4*e^4+2*C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(
1/2)*f+f)/(f*x+e))*x^2*f^4+C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2
*x^2+1)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-(d^2*e^2-f^2)/f^2)^
(1/2)*(-d^2*x^2+1)^(1/2)*f+f)/(f*x+e))*e^2*f^2)*csgn(d)^2*(d*x+1)^(1/2)*(-d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.42969, size = 3105, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
```

```
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.15 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.138713, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1609, 1809, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x\left(-2c-3ad^2-3bd^2x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}
\end{aligned}
$$

Mathematica [A] time = 0.0605943, size = 57, normalized size = 0.72

$$
\frac{3bd\sin^{-1}(dx)-\sqrt{1-d^2x^2}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)}{6d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*d*ArcSi
n[d*x])/(6*d^4)
```

Maple [C] time = 0., size = 139, normalized size = 1.8

$$
-\frac{\mathrm{csgn}\,(d)}{6\,d^4}\sqrt{-dx+1}\sqrt{dx+1}\Bigg(2\,\mathrm{csgn}\,(d)\,x^2cd^2\sqrt{-d^2x^2+1}+3\,\sqrt{-d^2x^2+1}\mathrm{csgn}\,(d)\,xbd^2+6\,\mathrm{csgn}\,(d)\,\sqrt{-d^2x^2+1}ad^2+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*x*b*d^2+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 4.96562, size = 134, normalized size = 1.7

$$
-\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}a}{d^2}+\frac{b\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\,\sqrt{d^2}d^2}-\frac{2\,\sqrt{-d^2x^2+1}c}{3\,d^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 2/3*sq
```

```
rt(-d^2*x^2 + 1)*c/d^4
```

Fricas [A] time = 1.14215, size = 189, normalized size = 2.39

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [C] time = 46.387, size = 313, normalized size = 3.96

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0, & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - a*meijerg(((-1, -3/4, -1/2, -1/4,
0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x
**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*m
eijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*c*meijerg(((-5
/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d*
*2*x**2))/(4*pi**(3/2)*d**4) - c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()
), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

Giac [A] time = 2.26382, size = 123, normalized size = 1.56

$$\frac{6\,bd^{10}\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)-\left(6\,ad^{11}-3\,bd^{10}+6\,cd^9+\left(2\,(dx+1)cd^9+3\,bd^{10}-4\,cd^9\right)(dx+1)\right)\sqrt{dx+1}\sqrt{-dx+1}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(6*b*d^10*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^11 - 3*b*d^10 +
 6*c*d^9 + (2*(d*x + 1)*c*d^9 + 3*b*d^10 - 4*c*d^9)*(d*x + 1))*sqrt(d*x + 1
)*sqrt(-d*x + 1))/d
```

3.16 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.060904, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.0322668, size = 45, normalized size = 0.71

$$\frac{(2ad^2+c)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

Maple [C] time = 0., size = 117, normalized size = 1.9

$$-\frac{\operatorname{csgn}(d)}{2\,d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(\operatorname{csgn}(d)\,d\sqrt{-d^2x^2+1}xc - 2\arctan\left(\frac{\operatorname{csgn}(d)\,dx}{\sqrt{-d^2x^2+1}}\right)ad^2 + 2\operatorname{csgn}(d)\,d\sqrt{-d^2x^2+1}b - \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A] time = 2.39676, size = 105, normalized size = 1.67

$$\frac{a\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt
(-d^2*x^2 + 1)*b/d^2 + 1/2*c*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.04285, size = 167, normalized size = 2.65

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [C] time = 20.8624, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\cdots\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \cdots\end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 1.86588, size = 97, normalized size = 1.54

$$-\frac{\left((dx+1)cd^4+2\,bd^5-cd^4\right)\sqrt{dx+1}\sqrt{-dx+1}-2\left(2\,ad^6+cd^4\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(((d*x + 1)*c*d^4 + 2*b*d^5 - c*d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
2*(2*a*d^6 + c*d^4)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.17 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rubi [A] time = 0.183471, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1809, 844, 216, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} - \frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right) \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0516867, size = 48, normalized size = 1.

$$
-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{b\sin^{-1}(dx)}{d} - \frac{c\sqrt{1-d^2x^2}}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Maple [C] time = 0., size = 96, normalized size = 2.

$$
\frac{\mathrm{csgn}(d)}{d^2}\left(-\mathrm{csgn}(d)\,\mathrm{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)ad^2 - \mathrm{csgn}(d)\sqrt{-d^2x^2+1}c + \arctan\left(\mathrm{csgn}(d)\,dx\frac{1}{\sqrt{-(dx+1)(dx-1)}}\right)b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+
arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2))*b*d)*(-d*x+1)^(1/2)*(d*x+1)^(1
/2)/d^2*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 4.22597, size = 89, normalized size = 1.85

$$-a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d^2*x/sqrt(d^2))/
sqrt(d^2) - sqrt(-d^2*x^2 + 1)*c/d^2
```

Fricas [A] time = 1.18311, size = 196, normalized size = 4.08

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] time = 28.2392, size = 245, normalized size = 5.1

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG_{6,6}^{2,6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
```

```
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.18 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rubi [A] time = 0.175533, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1807, 844, 216, 266, 63, 208}

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.056147, size = 48, normalized size = 1.

$$
-\frac{a\sqrt{1-d^2x^2}}{x} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{c\sin^{-1}(dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Maple [C] time = 0., size = 97, normalized size = 2.

$$
\frac{\text{csgn}(d)}{dx}\left(-\text{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\text{csgn}(d)\,dxb - \text{csgn}(d)\,d\sqrt{-d^2x^2+1}a + \arctan\left(\text{csgn}(d)\,dx\frac{1}{\sqrt{-d^2x^2+1}}\right)xc\right)\sqrt{-d
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-arctanh(1/(-d^2*x^2+1)^(1/2))*csgn(d)*d*x*b-csgn(d)*d*(-d^2*x^2+1)^(1/2)*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x*c)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/d/x
```

Maxima [A] time = 3.26579, size = 89, normalized size = 1.85

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d^2*x/sqrt(d^2))/
sqrt(d^2) - sqrt(-d^2*x^2 + 1)*a/x
```

Fricas [A] time = 1.14684, size = 201, normalized size = 4.19

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] time = 27.7528, size = 221, normalized size = 4.6

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{2,6}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
```

```
2))/(4*pi**(3/2)*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError
```

3.19 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rubi [A] time = 0.184015, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1609, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0466222, size = 56, normalized size = 0.79

$$
-\frac{\sqrt{1-d^2x^2}(a+2bx)}{2x^2}-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a + 2*b*x)*Sqrt[1 - d^2*x^2])/(2*x^2) - ((2*c + a*d^2)*ArcTanh[Sqrt[1 -
d^2*x^2]])/2
```

Maple [C] time = 0., size = 108, normalized size = 1.5

$$
-\frac{(\mathrm{csgn}\,(d))^2}{2\,x^2}\sqrt{-dx+1}\sqrt{dx+1}\left(\mathrm{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)x^2ad^2+2\,\mathrm{Artanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)x^2c+2\,\sqrt{-d^2x^2+1}xb+\sqrt{-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
x^2*a*d^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*x^2*c+2*(-d^2*x^2+1)^(1/2)*x*b+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Maxima [A] time = 3.97374, size = 132, normalized size = 1.86

$$-\frac{1}{2}\,ad^{2}\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^{2}x^{2}+1}b}{x}-\frac{\sqrt{-d^{2}x^{2}+1}a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Fricas [A] time = 1.01834, size = 154, normalized size = 2.17

$$\frac{\left(ad^{2}+2\,c\right)x^{2}\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\left(2\,bx+a\right)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [C] time = 34.2892, size = 218, normalized size = 3.07

$$\frac{iad^{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{ad^{2}G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{bdG}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) - a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1
), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) + I*b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)) + I*c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1
, 5/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - c*meijerg(((0, 1/4, 1/2,
 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*
x**2))/(4*pi**(3/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError
```

3.20 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=591

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}\left(8a^2Cf^2-b^2\left(3Ce^2-7f(2Af+Be)\right)\right)}{70b^4f}-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3b^2fx\right.}{}$$

```
[Out] ((A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2) + a^2*(a^2*f^2*(3*C*e + B*f) + 2*b^2*e^2*
(C*e + 3*B*f)))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) - ((8*a^2*C*f^2
 - b^2*(3*C*e^2 - 7*f*(B*e + 2*A*f)))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)^2*(a^2 - b^2*x^2))/(70*b^4*f) + ((3*C*e - 7*B*f)*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(42*b^2*f) - (C*Sqrt[a + b*x]*Sqrt[
a*c - b*c*x]*(e + f*x)^4*(a^2 - b^2*x^2))/(7*b^2*f) - (Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]*(8*(8*a^4*C*f^4 + 2*a^2*b^2*f^2*(15*C*e^2 + 7*f*(3*B*e + A*f))
- b^4*e^2*(3*C*e^2 - 7*f*(B*e + 12*A*f))) + 3*b^2*f*(a^2*f^2*(41*C*e + 35*B
*f) - 2*b^2*e*(3*C*e^2 - 7*f*(B*e + 7*A*f)))*x)*(a^2 - b^2*x^2))/(840*b^6*f
) + (a^2*Sqrt[c]*(A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2) + a^2*(a^2*f^2*(3*C*e + B
*f) + 2*b^2*e^2*(C*e + 3*B*f)))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rubi [A] time = 1.5174, antiderivative size = 584, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 833, 780, 195, 217, 203}

$$\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}\left(-\frac{8a^2Cf^2}{b^2}-7f(2Af+Be)+3Ce^2\right)}{70b^2f}-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3b^2fx\left(a^2f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^
2*b^2*e*f^2))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((3*C*e^2 - (8*
a^2*C*f^2)/b^2 - 7*f*(B*e + 2*A*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*
x)^2*(a^2 - b^2*x^2))/(70*b^2*f) + ((3*C*e - 7*B*f)*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(42*b^2*f) - (C*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]*(e + f*x)^4*(a^2 - b^2*x^2))/(7*b^2*f) - (Sqrt[a + b*x]*Sqrt[a*c
 - b*c*x]*(8*(8*a^4*C*f^4 + 2*a^2*b^2*f^2*(15*C*e^2 + 7*f*(3*B*e + A*f)) -
b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f))) + 3*b^2*f*(a^2*f^2*(41*C*e + 35*B*f
) - b^2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x)*(a^2 - b^2*x^2))/(840*b^6*f) +
 (a^2*Sqrt[c]*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b
^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c
]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)^3\sqrt{a^2c-b^2cx^2}\left(A+Bx+Cx^2\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int}{}$$

$$= \frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f} - \frac{C\sqrt{a+bx}\sqrt{a}}{}$$

$$= -\frac{\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}{70b^4f}$$

$$= -\frac{\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}{70b^4f}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2ef^2\right)\right)x\sqrt{}}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2ef^2\right)\right)x\sqrt{}}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2ef^2\right)\right)x\sqrt{}}{16b^4}$$

Mathematica [A] time = 1.40802, size = 427, normalized size = 0.72

$$\frac{\sqrt{c(a-bx)}\left(\left(a^2-b^2x^2\right)\left(a^4b^2f\left(7f(32Af+96Be+15Bfx)+C\left(672e^2+315efx+64f^2x^2\right)\right)+2a^2b^4\left(7Af\left(120e^2+45e\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*((a^2 - b^2*x^2)*(128*a^6*C*f^3 + a^4*b^2*f*(7*f*(96*B*e
 + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2)) + 2*a^2*b^4*(
7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*e^2*f*x + 24*e*f^
2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^2*x^2 + 8*f^3*x^3)
) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3) + x*(7*B
*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*C*x*(35*e^3 + 84*e^2
*f*x + 70*e*f^2*x^2 + 20*f^3*x^3)))) + 210*a^(5/2)*b*(a^4*f^2*(3*C*e + B*f)
 + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a -
b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(1680*b^6*
(-a + b*x)*Sqrt[a + b*x])
```

Maple [B] time = 0.038, size = 1446, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/1680*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)*(-224*A*a^4*b^2*f^3*(b^2*c)^(1/2)*(
-c*(b^2*x^2-a^2))^(1/2)-560*B*a^2*b^4*e^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^
```

```
(1/2)+840*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^6*c*e^3+
105*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^6*b^2*c*f^3+210*C*
arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*b^4*c*e^3+240*C*x^6*b^
6*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+280*B*x^5*b^6*f^3*(b^2*c)^(1/2
)*(-c*(b^2*x^2-a^2))^(1/2)+840*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b
^6*e^3-128*C*a^6*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+1680*A*x^2*b^6*
e^2*f*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-64*C*x^2*a^4*b^2*f^3*(b^2*c)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)-630*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*
x*a^2*b^4*e*f^2+336*A*x^4*b^6*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+42
0*C*x^3*b^6*e^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+560*B*x^2*b^6*e^3*(b
^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-1680*A*a^2*b^4*e^2*f*(b^2*c)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2)-672*B*a^4*b^2*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^
(1/2)-630*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^4*e^2*f-210*C*x^
3*a^2*b^4*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-336*B*x^2*a^2*b^4*e*
f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-336*C*x^2*a^2*b^4*e^2*f*(b^2*c)^
(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-315*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)
*x*a^4*b^2*e*f^2-70*B*x^3*a^2*b^4*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2
)+1260*B*x^3*b^6*e^2*f*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-112*A*x^2*a^2
*b^4*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-672*C*a^4*b^2*e^2*f*(b^2*c)
^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+630*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a
^2))^(1/2))*a^4*b^4*c*e*f^2+840*C*x^5*b^6*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-
a^2))^(1/2)+630*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*b^4*
c*e^2*f+315*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^6*b^2*c*e*
f^2-105*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^4*b^2*f^3-210*C*(b^2*c
)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^4*e^3+1008*B*x^4*b^6*e*f^2*(b^2*c)
^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-48*C*x^4*a^2*b^4*f^3*(b^2*c)^(1/2)*(-c*(b^2
*x^2-a^2))^(1/2)+1008*C*x^4*b^6*e^2*f*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2
)+1260*A*x^3*b^6*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2))/(-c*(b^2*x^2
-a^2))^(1/2)/b^6/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45001, size = 2147, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/3360*(105*(6*B*a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)
*e^3 + 3*(C*a^6*b + 2*A*a^4*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6 - 5
60*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^
5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 -
```

```
336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70
*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*
f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)
*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*
a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f
^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(6*B*a^4*b^3*e^2
*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3 + 3*(C*a^6*b + 2*A*a^4*b
^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2
*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 - 560*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e
*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*
e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 - 336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^
2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B
*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B
*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^
4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b
^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b^6]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.21 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=451

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3fx\left(5a^2Cf^2-b^2\left(2Ce^2-2f(5Af+2Be)\right)\right)+8\left(2a^2f^2(Bf+2Ce)-b^2e\left(Ce^2-2f(\right.\right.\right.}{120b^4f}$$

```
[Out] ((2*A*(4*b^4*e^2 + a^2*b^2*f^2) + a^2*(a^2*C*f^2 + 2*b^2*e*(C*e + 2*B*f)))*
x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((C*e - 2*B*f)*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(10*b^2*f) - (C*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(6*b^2*f) - (Sqrt[a + b*x]
*Sqrt[a*c - b*c*x]*(8*(2*a^2*f^2*(2*C*e + B*f) - b^2*e*(C*e^2 - 2*f*(B*e +
5*A*f))) + 3*f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2*f*(2*B*e + 5*A*f)))*x)*(a^2
- b^2*x^2))/(120*b^4*f) + (a^2*Sqrt[c]*(2*A*(4*b^4*e^2 + a^2*b^2*f^2) + a^2
*(a^2*C*f^2 + 2*b^2*e*(C*e + 2*B*f)))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTa
n[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rubi [A] time = 1.00967, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 833, 780, 195, 217, 203}

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3fx\left(5a^2Cf^2-b^2\left(2Ce^2-2f(5Af+2Be)\right)\right)+8\left(2a^2f^2(Bf+2Ce)-\frac{1}{8}b^2\left(8Ce^3-16\right.\right.\right.}{120b^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*x*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((C*e - 2*B*f)*Sqrt[a + b*x]*Sq
rt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(10*b^2*f) - (C*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(6*b^2*f) - (Sqrt[a + b*x]*S
qrt[a*c - b*c*x]*(8*(2*a^2*f^2*(2*C*e + B*f) - (b^2*(8*C*e^3 - 16*e*f*(B*e
+ 5*A*f)))/8) + 3*f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2*f*(2*B*e + 5*A*f)))*x)*
(a^2 - b^2*x^2))/(120*b^4*f) + (a^2*Sqrt[c]*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e +
 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*Ar
cTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2
])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
```

```
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)^2\sqrt{a^2c-b^2cx^2}\left(A+Bx+Cx^2\right)d}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \left(\sqrt{a+bx}\sqrt{ac-bc}\right.$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - C\sqrt{a+bx}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - C\sqrt{a+bx}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a+bx}\sqrt{}}{16b^4}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a+bx}\sqrt{}}{16b^4}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a+bx}\sqrt{}}{16b^4}$$

Mathematica [A] time = 0.995131, size = 311, normalized size = 0.69

$$\sqrt{c(a-bx)}\left(b\left(a^2-b^2x^2\right)\left(2a^2b^2\left(5Af(16e+3fx)+B\left(40e^2+30efx+8f^2x^2\right)+Cx\left(15e^2+16efx+5f^2x^2\right)\right)+a^4f(3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*(a^2 - b^2*x^2)*(a^4*f*(64*C*e + 32*B*f + 15*C*f*x) +
 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x + 5*f^2*x^2) + B*
(40*e^2 + 30*e*f*x + 8*f^2*x^2)) - 4*b^4*x*(5*A*(6*e^2 + 8*e*f*x + 3*f^2*x^
2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^2 + 24*e*f*x + 10*f
^2*x^2)))) + 30*a^(5/2)*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4
*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(
Sqrt[2]*Sqrt[a])]))/(240*b^5*(-a + b*x)*Sqrt[a + b*x])
```

Maple [B] time = 0.017, size = 987, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/240*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)*(120*A*arctan((b^2*c)^(1/2)*x/(-c*(b
^2*x^2-a^2))^(1/2))*a^2*b^4*c*e^2+30*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-
a^2))^(1/2))*a^4*b^2*c*e^2+120*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b
^4*e^2-15*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^4*f^2+40*C*x^5*b^4*f
^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+48*B*x^4*b^4*f^2*(-c*(b^2*x^2-a^2
))^(1/2)*(b^2*c)^(1/2)+60*A*x^3*b^4*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1
```

```
/2)+60*C*x^3*b^4*e^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+80*B*x^2*b^4*e^
2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)-80*B*a^2*b^2*e^2*(-c*(b^2*x^2-a^2)
)^(1/2)*(b^2*c)^(1/2)-32*B*a^4*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+1
5*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^6*c*f^2-32*C*x^2*a^2
*b^2*e*f*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)-64*C*a^4*e*f*(-c*(b^2*x^2-a
^2))^(1/2)*(b^2*c)^(1/2)+30*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/
2))*a^4*b^2*c*f^2-160*A*a^2*b^2*e*f*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+
60*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*b^2*c*e*f-30*A*(b
^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^2*f^2-30*C*(b^2*c)^(1/2)*(-c*(
b^2*x^2-a^2))^(1/2)*x*a^2*b^2*e^2+96*C*x^4*b^4*e*f*(-c*(b^2*x^2-a^2))^(1/2)
*(b^2*c)^(1/2)+120*B*x^3*b^4*e*f*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)-10*
C*x^3*a^2*b^2*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+160*A*x^2*b^4*e*f*
(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)-16*B*x^2*a^2*b^2*f^2*(-c*(b^2*x^2-a^
2))^(1/2)*(b^2*c)^(1/2)-60*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b
^2*e*f)/(-c*(b^2*x^2-a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30742, size = 1517, normalized size = 3.36

$$\left[\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/480*(15*(4*B*a^4*b^2*e*f + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*
A*a^4*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^
4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f
 - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(
5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^
2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*s
qrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*e*f + 2*(C*a^
4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*A*a^4*b^2)*f^2)*sqrt(c)*arctan(sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40*C*b^5*f^2
*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4
 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*
C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3
 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 +
(C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)^{2}\left(A+Bx+Cx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2*(A + B*x + C*x**2),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.22 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=300

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(4\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)-3b^2fx(3Ce-5Bf)\right)}{60b^4f}+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}{}$$

```
[Out] ((4*A*b^2*e + a^2*(C*e + B*f))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(8*b^2) -
 (C*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(5*b^2*f)
- (Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(4*(2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*
e + A*f))) - 3*b^2*f*(3*C*e - 5*B*f)*x)*(a^2 - b^2*x^2))/(60*b^4*f) + (a^2*
Sqrt[c]*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTa
n[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rubi [A] time = 0.445962, antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1610, 1654, 780, 195, 217, 203}

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(4\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)-3b^2fx(3Ce-5Bf)\right)}{60b^4f}+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A*e + (a^2*(C*e + B*f))/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (C*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(5*b^2*f) - (S
qrt[a + b*x]*Sqrt[a*c - b*c*x]*(4*(2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e +
A*f))) - 3*b^2*f*(3*C*e - 5*B*f)*x)*(a^2 - b^2*x^2))/(60*b^4*f) + (a^2*Sqrt
[c]*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)\sqrt{a^2c-b^2cx^2}\left(A+Bx+Cx^2\right)\,dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right.}{}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}\,(}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-b}}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-b}}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-b}}{}$$

Mathematica [A] time = 0.646006, size = 200, normalized size = 0.67

$$-\frac{c\left(\left(a^2-b^2x^2\right)\left(a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))+16a^4Cf-2b^4x(10A(3e+2fx)+x(5B(4e+3fx)+3\right.\right.}{120b^4\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] -(c*((a^2 - b^2*x^2)*(16*a^4*C*f + a^2*b^2*(40*A*f + 5*B*(8*e + 3*f*x) + C*
x*(15*e + 8*f*x)) - 2*b^4*x*(10*A*(3*e + 2*f*x) + x*(5*B*(4*e + 3*f*x) + 3*
```

```
C*x*(5*e + 4*f*x)))) + 30*a^(5/2)*b*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a -
b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(120*b^4*S
qrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [B] time = 0.013, size = 588, normalized size = 2.

$$\frac{1}{120\,b^4}\sqrt{bx+a}\sqrt{-c\,(bx-a)}\left(24\,Cx^4b^4f\sqrt{b^2c}\sqrt{-c\left(b^2x^2-a^2\right)}+30\,Bx^3b^4f\sqrt{b^2c}\sqrt{-c\left(b^2x^2-a^2\right)}+30\,Cx^3b^4e\sqrt{b^2c}\sqrt{-c\left(b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)*(24*C*x^4*b^4*f*(b^2*c)^(1/2)*(-c*(b
^2*x^2-a^2))^(1/2)+30*B*x^3*b^4*f*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+30
*C*x^3*b^4*e*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+60*A*arctan((b^2*c)^(1/
2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^4*c*e+40*A*x^2*b^4*f*(b^2*c)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2)+15*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))
*a^4*b^2*c*f+40*B*x^2*b^4*e*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+15*C*arc
tan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*b^2*c*e-8*C*x^2*a^2*b^2*f
*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+60*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2
))^(1/2)*x*b^4*e-15*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^2*f-15
*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^2*e-40*A*a^2*b^2*f*(b^2*c
)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-40*B*a^2*b^2*e*(b^2*c)^(1/2)*(-c*(b^2*x^2-
a^2))^(1/2)-16*C*a^4*f*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2))/(-c*(b^2*x^2
-a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27366, size = 980, normalized size = 3.27

$$\left[\frac{15\left(Ba^4bf+\left(Ca^4b+4\,Aa^2b^3\right)e\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(24\,Cb^4fx^4-40\,Ba^2b^2e+3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2
 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f
```

```
*x^4 - 40*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*
b^2 - 5*A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*
a^2*b^2 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120*(15*
(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 - 40*B*a^2*
b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*b^2 - 5*A*b^4)*f
)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*a^2*b^2 - 4*A*b^
4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c(-a+bx)}\sqrt{a+bx}\left(e+fx\right)\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.23 $\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=221

$$\frac{a^2\sqrt{c}\sqrt{a+bx}\left(a^2C+4Ab^2\right)\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\left(a^2-b^2x^2\right)}{3b^2}$$

```
[Out] ((4*A + (a^2*C)/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (B*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(a^2 - b^2*x^2))/(3*b^2) - (C*x*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(a^2 - b^2*x^2))/(4*b^2) + (a^2*Sqrt[c]*(4*A*b^2 + a^2*C)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b
^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rubi [A] time = 0.146967, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {901, 1815, 641, 195, 217, 203}

$$\frac{a^2\sqrt{c}\sqrt{a+bx}\left(a^2C+4Ab^2\right)\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\left(a^2-b^2x^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A + (a^2*C)/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (B*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(a^2 - b^2*x^2))/(3*b^2) - (C*x*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(a^2 - b^2*x^2))/(4*b^2) + (a^2*Sqrt[c]*(4*A*b^2 + a^2*C)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b
^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int \sqrt{a^2c-b^2cx^2}\left(A+Bx+Cx^2\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{4b^2} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\left(-c\left(4Ab^2+\right.\right.}{4b^2c\sqrt{a^2c}}$$

$$= -\frac{B\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{3b^2} - \frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{4b^2} + $$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{3b^2} - $$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{3b^2} - $$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\sqrt{ac-bcx}\left(a^2-b^2x^2\right)}{3b^2} - $$

Mathematica [A] time = 0.380484, size = 142, normalized size = 0.64

$$-\frac{c\left(b\left(b^2x^2-a^2\right)\left(2b^2x\left(6A+4Bx+3Cx^2\right)-a^2(8B+3Cx)\right)+6a^{5/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\left(a^2C+4Ab^2\right)\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\right)}{24b^3\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] -(c*(b*(-a^2 + b^2*x^2)*(-(a^2*(8*B + 3*C*x)) + 2*b^2*x*(6*A + 4*B*x + 3*C*
x^2)) + 6*a^(5/2)*(4*A*b^2 + a^2*C)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[
Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(24*b^3*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [A] time = 0.011, size = 287, normalized size = 1.3

$$\frac{1}{24b^2}\sqrt{bx+a}\sqrt{-c(bx-a)}\left(6Cx^3b^2\sqrt{-c\left(b^2x^2-a^2\right)}\sqrt{b^2c}+12A\arctan\left(\frac{\sqrt{b^2c}x}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)a^2b^2c+8Bx^2b^2\sqrt{-c\left(b^2x^{\dots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)*(6*C*x^3*b^2*(-c*(b^2*x^2-a^2))^(1/2)
*(b^2*c)^(1/2)+12*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^
2*c+8*B*x^2*b^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)+3*C*arctan((b^2*c)^(
1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*c+12*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2)
)^(1/2)*x*b^2-3*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2-8*B*a^2*(-c*
(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2))/(-c*(b^2*x^2-a^2))^(1/2)/b^2/(b^2*c)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.14845, size = 602, normalized size = 2.72

$$\left[\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\right.\right.}{48\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a
*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*
B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3, -
1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B*a^2*
b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.24 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C\left(a^2-\right.}{b^2f\sqrt{a+bx}}$$

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.489579, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 844, 217, 203, 725, 204}

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C\left(a^2-\right.}{b^2f\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx &= \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\left(\left(Ce^2-Be\right.\right.}{} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b}}\right.}{f^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\left(Ce^2-Be\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.786412, size = 225, normalized size = 0.81

$$\frac{\sqrt{a-bx}\left(\frac{2\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{af-be}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)(aCf-bBf+bCe)}{b^2}+\frac{Cf\sqrt{a+bx}\left(-\sqrt{a-bx}-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{b^2}\right)}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] (Sqrt[a - b*x]*((C*f*Sqrt[a + b*x]*(-Sqrt[a - b*x] - (2*Sqrt[a]*ArcSin[Sqrt
[a - b*x]/(Sqrt[2]*Sqrt[a])])/Sqrt[1 + (b*x)/a]))/b^2 + (2*(b*C*e - b*B*f +
 a*C*f)*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/b^2 + (2*(C*e^2 + f*(-(B*e) +
A*f))*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a
+ b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[-(b*e) + a*f])))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0.054, size = 503, normalized size = 1.8

$$\frac{1}{b^2f^3c}\left(-A\ln\left(2\frac{1}{fx+e}\left(b^2cex+a^2cf+\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{-c\left(b^2x^2-a^2\right)}f\right)\right)b^2cf^2\sqrt{b^2c}+B\ln\left(2\frac{1}{fx+e}\left(b^2cex+a^2cf\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (-A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^
2))^(1/2)*f)/(f*x+e))*b^2*c*f^2*(b^2*c)^(1/2)+B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e*f
*(b^2*c)^(1/2)+B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*f^2
*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*(b^2*c)^(1/2
)-C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*e*f*(c*(a^2*f^2-
b^2*e^2)/f^2)^(1/2)-C*f^2*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2))*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/b^2/(c*(a^2*f^2-b^2
*e^2)/f^2)^(1/2)/(b^2*c)^(1/2)/f^3/c/(-c*(b^2*x^2-a^2))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\sqrt{-c(-a + bx)}\sqrt{a + bx}\,(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.25 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

Optimal. Leaf size=322

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right.}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^{3/2}}$$

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rubi [A] time = 0.57944, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1651, 844, 217, 203, 725, 204}

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right.}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{c\left(Ab^2e+a^2(Ce-Bf)\right)+cC\left(\frac{b^2}{}\right.}{(e+fx)\sqrt{a^2c-b^2cx^2}}}{c\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{}{\sqrt{a^2c}}}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-b}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{cx}}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} +$$

Mathematica [A] time = 1.03278, size = 309, normalized size = 0.96

$$\frac{\frac{2b^2e\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(af-be)^{3/2}} + \frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)(af-be)(af+be)} - \frac{2\sqrt{a-bx}(2Ce-Bf)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{af-be}} - \frac{2C\sqrt{a-bx}\tan^{-1}\left(\right.}{b}}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)) - (2*C*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x
]])/b - (2*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a -
b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/(Sqrt[-(b*e) - a*f]*Sqrt[-(b*e)
+ a*f]) + (2*b^2*e*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*ArcTan[(Sqrt[-(
b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) -
a*f)^(3/2)*(-(b*e) + a*f)^(3/2)))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0.046, size = 1200, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2
))^(1/2)*f)/(f*x+e))*x*b^2*c*e*f^3*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*
c*f^4*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*c*e*f^3*(b^2*c)^(1/2)-C*ln(
2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/
2)*f)/(f*x+e))*x*b^2*c*e^3*f*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(-c*(b^
2*x^2-a^2))^(1/2))*x*a^2*c*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^
2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*x*b^2*c*e^2*f^2*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(
b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^
(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^
2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/
(-c*(b^2*x^2-a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arc
tan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e
^2)/f^2)^(1/2)-A*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2
*e^2)/f^2)^(1/2)+B*e*f^3*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a^2*f^2
-b^2*e^2)/f^2)^(1/2)-C*e^2*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a
^2*f^2-b^2*e^2)/f^2)^(1/2))/c*(-c*(b*x-a))^(1/2)*(b*x+a)^(1/2)/(-c*(b^2*x^2
-a^2))^(1/2)/(a*f+b*e)/(a*f-b*e)/(f*x+e)/(b^2*c)^(1/2)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.26 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{(a^2-b^2x^2)\left(2a^2f^2(2Ce-Bf)-b^2e\left(f(Be-3Af)+Ce^2\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}}{}$$

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*e*
(C*e^2 + f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((A*(2*b^4*e^2 + a^2*b^2*f^2) + a^
2*(2*a^2*C*f^2 + b^2*e*(C*e - 3*B*f)))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt
[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/
(2*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.676915, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1610, 1651, 807, 725, 204}

$$\frac{(a^2-b^2x^2)\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{2c\left(Ab^2e+a^2(Ce-Bf)\right)-c\ldots}{(e+fx)^2\ldots}}{2c\left(b^2e^2-a^2f^2\right)\sqrt{a+\ldots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

Mathematica [A] time = 1.91553, size = 492, normalized size = 1.36

$$\frac{\frac{b^2\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\left(2(e+fx)\left(a^2f^2+2b^2e^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)+3ef\sqrt{a-bx}\sqrt{a+bx}\sqrt{-af-be}\sqrt{af-be}\right)}{(e+fx)(-af-be)^{5/2}(af-be)^{5/2}}+\frac{2f(bx-a)\sqrt{a+bx}(Bf-2Ce)}{(e+fx)\left(a^2f^2-b^2e^2\right)}+\frac{f(bx-a)\sqrt{a+\ldots}}{(e+fx)^2\ldots}\ldots}{2f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)^2) + (2*f*(-2*C*e + B*f)*(-a + b*x)*Sqrt[a + b*x])/((-(
b^2*e^2) + a^2*f^2)*(e + f*x)) + (4*C*Sqrt[a - b*x]*ArcTan[(Sqrt[-(b*e) + a
*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/(Sqrt[-(b*e) - a*f]
```

```
*Sqrt[-(b*e) + a*f]) - (4*b^2*e*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTan[(Sqrt[-(
b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) -
a*f)^(3/2)*(-(b*e) + a*f)^(3/2)) + (b^2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a -
 b*x]*(3*e*f*Sqrt[-(b*e) - a*f]*Sqrt[-(b*e) + a*f]*Sqrt[a - b*x]*Sqrt[a + b
*x] + 2*(2*b^2*e^2 + a^2*f^2)*(e + f*x)*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a -
 b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]]))/((-(b*e) - a*f)^(5/2)*(-(b*e)
+ a*f)^(5/2)*(e + f*x)))/(2*f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0.052, size = 1848, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2*(C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^
2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*e^2*f^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f
+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2
*b^2*c*e*f^3-6*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-
c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*b^2*c*e^2*f^2+2*C*ln(2*(b^2*c*e*x+
a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e)
)*x*a^2*b^2*c*e^3*f+A*a^2*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*e*f^3+2*A*ln(2*(b^2*c*e*x+a
^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))
*b^4*c*e^4+4*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*
(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^4*c*e*f^3+A*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*
c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(
b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^3*f-3*A*x*b^2*e*f^3*(-c*(b^2*x^
2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*x*b^2*e^2*f^2*(-c*(b^2*x^2-
a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*C*x*a^2*e*f^3*(-c*(b^2*x^2-a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+C*x*b^2*e^3*f*(-c*(b^2*x^2-a^2))^
(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2
-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*f^4
+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a
^2))^(1/2)*f)/(f*x+e))*x^2*b^4*c*e^2*f^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^
2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*b^4*c*e^3*
f+2*B*x*a^2*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*
A*b^2*e^2*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*a^
2*e*f^3*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^2*e^
3*f*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^2*e^2*f^
2*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*ln(2*(b^2*c*
e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*
x+e))*x^2*a^4*c*f^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a
^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))
*a^2*b^2*c*e^4)/c*(-c*(b*x-a))^(1/2)*(b*x+a)^(1/2)/(-c*(b^2*x^2-a^2))^(1/2)
/(a*f+b*e)/(a*f-b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 16.9052, size = 2238, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*C*a^4*sqrt(-c)*c^2*f^2 + A*a^2*b^2*sqrt(-c)*c^2*f^2 - 3*B*a^2*b^2*sqrt(
-c)*c^2*f*e + C*a^2*b^2*sqrt(-c)*c^2*e^2 + 2*A*b^4*sqrt(-c)*c^2*e^2)*arctan
(1/2*(2*b*c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*
c)*c))^2*f)/(sqrt(a^2*f^2 - b^2*e^2)*c^2))/((a^4*f^4*abs(c) - 2*a^2*b^2*f^2
*abs(c)*e^2 + b^4*abs(c)*e^4)*sqrt(a^2*f^2 - b^2*e^2)*c^2) + 2*(16*B*a^6*b*
sqrt(-c)*c^8*f^5 - 32*C*a^6*b*sqrt(-c)*c^8*f^4*e - 24*A*a^4*b^3*sqrt(-c)*c^
8*f^4*e + 4*A*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2*sqrt(-c)*c^6*f^5 + 8*B*a^4*b^3*sqrt(-c)*c^8*f^3*e^2 + 20*B*a^4
*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt
(-c)*c^6*f^4*e + 4*B*a^4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b
*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^5 + 8*C*a^4*b^3*sqrt(-c)*c^8*f^2*e^3 - 44*
C*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2
*sqrt(-c)*c^6*f^3*e^2 - 40*A*a^2*b^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*
a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^3*e^2 - 8*C*a^4*b*(sqrt(-b*c*x +
```

```
a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^4*e - 6*
A*a^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4
*sqrt(-c)*c^4*f^4*e - A*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2
 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^5 + 16*B*a^2*b^4*(sqrt(-b*c*x + a*c)*
sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^2*e^3 + 10*B*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^3*e^2 + 3*B*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^
2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^4*e + 8*C*a^2*b^4*(sqrt(-b*c*x + a*c
)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f*e^4 - 14*C*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^2*e^3 - 12*A*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 +
 (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^2*e^3 - 5*C*a^2*b^2*(sqrt(-b*c*x + a*c)
*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^3*e^2 - 2*A*b
^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-
c)*c^2*f^3*e^2 + 4*B*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^4*sqrt(-c)*c^4*f*e^4 + 4*C*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) -
 sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*e^5 + 2*C*b^4*(sqrt(-b*c*x
 + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f*e^4)/(
(a^4*f^6*abs(c) - 2*a^2*b^2*f^4*abs(c)*e^2 + b^4*f^2*abs(c)*e^4)*(4*a^2*c^4
*f + 4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*
c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*f
)^2)
```

3.27 $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=501

$$-\frac{\left(a^2-b^2x^2\right)(e+fx)^2\left(16a^2Cf^2-b^2\left(3Ce^2-5f(4Af+3Be)\right)\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left(a^2-b^2x^2\right)\left(b^2fx\left(a^2f^2(45Bf+71Ce)-2b^2e\left(3Ce\right.\right.\right.}{}$$

```
[Out] -((16*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(3*B*e + 4*A*f)))*(e + f*x)^2*(a^2 - b
^2*x^2))/(60*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e - 5*B*f)*(e + f
*x)^3*(a^2 - b^2*x^2))/(20*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - (C*(e +
 f*x)^4*(a^2 - b^2*x^2))/(5*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - ((4*(1
6*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2 + 5*f*(3*B*e + A*f)) - b^4*e^2*(3*C*e
^2 - 5*f*(3*B*e + 16*A*f))) + b^2*f*(a^2*f^2*(71*C*e + 45*B*f) - 2*b^2*e*(3
*C*e^2 - 5*f*(3*B*e + 10*A*f)))*x)*(a^2 - b^2*x^2))/(120*b^6*f*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]) + ((4*A*(2*b^4*e^3 + 3*a^2*b^2*e*f^2) + a^2*(3*a^2*f^2
*(3*C*e + B*f) + 4*b^2*e^2*(C*e + 3*B*f)))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(
b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x])
```

Rubi [A] time = 1.28111, antiderivative size = 496, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1610, 1654, 833, 780, 217, 203}

$$\frac{\left(a^2-b^2x^2\right)(e+fx)^2\left(-\frac{16a^2Cf^2}{b^2}-5f(4Af+3Be)+3Ce^2\right)}{60b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left(a^2-b^2x^2\right)\left(b^2fx\left(a^2f^2(45Bf+71Ce)-b^2\left(6Ce^3-10ef\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((3*C*e^2 - (16*a^2*C*f^2)/b^2 - 5*f*(3*B*e + 4*A*f))*(e + f*x)^2*(a^2 - b^
2*x^2))/(60*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e - 5*B*f)*(e + f*
x)^3*(a^2 - b^2*x^2))/(20*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - (C*(e +
f*x)^4*(a^2 - b^2*x^2))/(5*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - ((4*(16
*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2 + 5*f*(3*B*e + A*f)) - b^4*e^2*(3*C*e^
2 - 5*f*(3*B*e + 16*A*f))) + b^2*f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^
3 - 10*e*f*(3*B*e + 10*A*f)))*x)*(a^2 - b^2*x^2))/(120*b^6*f*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]) + ((3*a^4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f
) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*e*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sq
rt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(-c\left(5Ab^2+4a^2C\right)f^2+b^2cf(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)}{}}{}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+}{20b^2f\sqrt{a+}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+}{20b^2f\sqrt{a+}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+}{20b^2f\sqrt{a+}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+}{20b^2f\sqrt{a+}}$$

Mathematica [B] time = 6.5421, size = 1107, normalized size = 2.21

$$-\frac{a^4Cf^3(a-bx)\sqrt{a+bx}\left(\frac{630\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a-bx}\left(2-\frac{a-bx}{a}\right)^{11/2}}+\frac{4}{1-\frac{a-bx}{2a}}+\frac{18}{\left(2-\frac{a-bx}{a}\right)^2}+\frac{42}{\left(2-\frac{a-bx}{a}\right)^3}+\frac{105}{\left(2-\frac{a-bx}{a}\right)^4}+\frac{315}{\left(2-\frac{a-bx}{a}\right)^5}\right)\left(2-\frac{a-bx}{a}\right)^{11/2}}{40b^6\sqrt{c(a-bx)}\sqrt{\frac{a+bx}{a}}} - \frac{a^3f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] -(a^4*C*f^3*(a - b*x)*Sqrt[a + b*x]*(2 - (a - b*x)/a)^(11/2)*(315/(2 - (a -
 b*x)/a)^5 + 105/(2 - (a - b*x)/a)^4 + 42/(2 - (a - b*x)/a)^3 + 18/(2 - (a
- b*x)/a)^2 + 4/(1 - (a - b*x)/(2*a)) + (630*Sqrt[a]*ArcSin[Sqrt[a - b*x]/(
Sqrt[2]*Sqrt[a])])/(Sqrt[a - b*x]*(2 - (a - b*x)/a)^(11/2))))/(40*b^6*Sqrt[
c*(a - b*x)]*Sqrt[(a + b*x)/a]) - (a^3*f^2*(3*b*C*e + b*B*f - 5*a*C*f)*(a -
 b*x)*Sqrt[a + b*x]*(2 - (a - b*x)/a)^(9/2)*(105/(2 - (a - b*x)/a)^4 + 35/(
2 - (a - b*x)/a)^3 + 14/(2 - (a - b*x)/a)^2 + 3/(1 - (a - b*x)/(2*a)) + (21
0*Sqrt[a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a - b*x]*(2 - (a -
 b*x)/a)^(9/2))))/(24*b^6*Sqrt[c*(a - b*x)]*Sqrt[(a + b*x)/a]) - (a^2*f*(10
*a^2*C*f^2 - 4*a*b*f*(3*C*e + B*f) + b^2*(3*C*e^2 + f*(3*B*e + A*f)))*(a -
b*x)*Sqrt[a + b*x]*(2 - (a - b*x)/a)^(7/2)*(15/(2 - (a - b*x)/a)^3 + 5/(2 -
 (a - b*x)/a)^2 + (1 - (a - b*x)/(2*a))^(-1) + (30*Sqrt[a]*ArcSin[Sqrt[a -
b*x]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a - b*x]*(2 - (a - b*x)/a)^(7/2))))/(6*b^6*S
qrt[c*(a - b*x)]*Sqrt[(a + b*x)/a]) - (a*(b*e - a*f)*(10*a^2*C*f^2 - 2*a*b*
f*(4*C*e + 3*B*f) + b^2*(C*e^2 + 3*f*(B*e + A*f)))*(a - b*x)*Sqrt[a + b*x]*
(2 - (a - b*x)/a)^(5/2)*(6/(2 - (a - b*x)/a)^2 + (1 - (a - b*x)/(2*a))^(-1)
 + (12*Sqrt[a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[a - b*x]*(2 -
 (a - b*x)/a)^(5/2))))/(4*b^6*Sqrt[c*(a - b*x)]*Sqrt[(a + b*x)/a]) - ((b*e
- a*f)^2*(5*a^2*C*f + b^2*(B*e + 3*A*f) - 2*a*b*(C*e + 2*B*f))*(a - b*x)*Sq
rt[a + b*x]*(2 - (a - b*x)/a)^(3/2)*((2 - (a - b*x)/a)^(-1) + (2*Sqrt[a]*Ar
```

```
cSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a - b*x]*(2 - (a - b*x)/a)^(3/
2))))/(b^6*Sqrt[c*(a - b*x)]*Sqrt[(a + b*x)/a]) - (2*(A*b^2 - a*(b*B - a*C)
)*(b*e - a*f)^3*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(b^6*Sqr
t[c*(a - b*x)])
```

Maple [B] time = 0.028, size = 965, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/c*(-24*C*x^4*b^4*f^3*(b^2*c)^(1/2)*(
-c*(b^2*x^2-a^2))^(1/2)-30*B*x^3*b^4*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(
1/2)-90*C*x^3*b^4*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+180*A*arctan
((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^4*c*e*f^2+120*A*arctan((b^
2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^6*c*e^3-40*A*x^2*b^4*f^3*(b^2*c)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)+45*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2)
)^(1/2))*a^4*b^2*c*f^3+180*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2
))*a^2*b^4*c*e^2*f-120*B*x^2*b^4*e*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/
2)+135*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*b^2*c*e*f^2+6
0*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^4*c*e^3-32*C*x^2
*a^2*b^2*f^3*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-120*C*x^2*b^4*e^2*f*(b^
2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)-180*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^
(1/2)*x*b^4*e*f^2-45*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^2*f^3
-180*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b^4*e^2*f-135*C*(b^2*c)^(1/
2)*(-c*(b^2*x^2-a^2))^(1/2)*x*a^2*b^2*e*f^2-60*C*(b^2*c)^(1/2)*(-c*(b^2*x^2
-a^2))^(1/2)*x*b^4*e^3-80*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*a^2*b^2*
f^3-360*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*b^4*e^2*f-240*B*(b^2*c)^(1
/2)*(-c*(b^2*x^2-a^2))^(1/2)*a^2*b^2*e*f^2-120*B*(b^2*c)^(1/2)*(-c*(b^2*x^2
-a^2))^(1/2)*b^4*e^3-64*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*a^4*f^3-24
0*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*a^2*b^2*e^2*f)/b^6/(-c*(b^2*x^2-
a^2))^(1/2)/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11801, size = 1538, normalized size = 3.07

$$\left[-\frac{15\left(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\left(Ca^2b^3+2\,Ab^5\right)e^3+3\left(3\,Ca^4b+4\,Aa^2b^3\right)ef^2\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/240*(15*(12*B*a^2*b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e
^3 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 + 12
0*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*
(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*
b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4
*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*
x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(12*B*a^2*b^3*e^2*
f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e^3 + 3*(3*C*a^4*b + 4*A*a^2*b^
3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*
c*x^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 120*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 +
 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3
*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a
^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2
*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)
)/(b^6*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.28 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=368

$$-\frac{(a^2-b^2x^2)\left(fx\left(9a^2Cf^2-b^2\left(2Ce^2-4f(3Af+2Be)\right)\right)+4\left(4a^2f^2(Bf+2Ce)-b^2e\left(Ce^2-4f(3Af+Be)\right)\right)\right)}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-}}{}$$

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*(a^2 - b^2*x^2))/(12*b^2*f*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]) - (C*(e + f*x)^3*(a^2 - b^2*x^2))/(4*b^2*f*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]) - ((4*(4*a^2*f^2*(2*C*e + B*f) - b^2*e*(C*e^2 - 4*f*(B*e + 3*A
*f))) + f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*f)))*x)*(a^2 - b^2
*x^2))/(24*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((4*A*(2*b^4*e^2 + a^2*
b^2*f^2) + a^2*(3*a^2*C*f^2 + 4*b^2*e*(C*e + 2*B*f)))*Sqrt[a^2*c - b^2*c*x^
2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b
*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.875056, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1610, 1654, 833, 780, 217, 203}

$$-\frac{(a^2-b^2x^2)\left(fx\left(9a^2Cf^2-b^2\left(2Ce^2-4f(3Af+2Be)\right)\right)+4\left(4a^2f^2(Bf+2Ce)-\frac{1}{4}b^2\left(4Ce^3-16ef(3Af+Be)\right)\right)\right)}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*(a^2 - b^2*x^2))/(12*b^2*f*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]) - (C*(e + f*x)^3*(a^2 - b^2*x^2))/(4*b^2*f*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]) - ((4*(4*a^2*f^2*(2*C*e + B*f) - (b^2*(4*C*e^3 - 16*e*f*(B*e +
 3*A*f)))/4) + f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*f)))*x)*(a^
2 - b^2*x^2))/(24*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((3*a^4*C*f^2 +
4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2
*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[
a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(-c\left(4Ab^2+3a^2C\right)f^2+b^2cf(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{4b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)}{}}{}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)\right.\right.}{}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)\right.\right.}{}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)\right.\right.}{}$$

Mathematica [A] time = 3.81353, size = 555, normalized size = 1.51

$$-12\sqrt{a-bx}\sqrt{a+bx}\left(6a^{3/2}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a-bx}(4a+bx)\sqrt{\frac{bx}{a}+1}\right)\left(6a^2Cf^2-3abf(Bf+2Ce)+b^2\left(f(Af+2Be)+C\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-24*(b*e - a*f)*(4*a^2*C*f + b^2*(B*e + 2*A*f) - a*b*(2*C*e + 3*B*f))*Sqrt
[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a] + 2*Sqrt[a]*ArcSin
[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 12*(6*a^2*C*f^2 - 3*a*b*f*(2*C*e + B*f
) + b^2*(C*e^2 + f*(2*B*e + A*f)))*Sqrt[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*
x]*(4*a + b*x)*Sqrt[1 + (b*x)/a] + 6*a^(3/2)*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*
Sqrt[a])]) - 4*f*(2*b*C*e + b*B*f - 4*a*C*f)*Sqrt[a - b*x]*Sqrt[a + b*x]*(S
qrt[a - b*x]*Sqrt[1 + (b*x)/a]*(22*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^(5/2)*
ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - C*f^2*Sqrt[a + b*x]*((a - b*x)*S
qrt[1 + (b*x)/a]*(160*a^3 + 81*a^2*b*x + 32*a*b^2*x^2 + 6*b^3*x^3) + 210*a^
(7/2)*Sqrt[a - b*x]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 48*(A*b^2 +
a*(-(b*B) + a*C))*(b*e - a*f)^2*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcTan[Sqrt
[a - b*x]/Sqrt[a + b*x]])/(24*b^5*Sqrt[c*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.025, size = 635, normalized size = 1.7

$$\frac{1}{24\,cb^4}\sqrt{bx+a}\sqrt{-c\,(bx-a)}\left(-6\,Cx^3b^2f^2\sqrt{b^2c}\sqrt{-c\left(b^2x^2-a^2\right)}+12\,A\arctan\left(\frac{\sqrt{b^2c}x}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)a^2b^2cf^2+24\,A\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/c*(-6*C*x^3*b^2*f^2*(b^2*c)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2)+12*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))
*a^2*b^2*c*f^2+24*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^4*c*
e^2+24*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^2*c*e*f-8*B
*x^2*b^2*f^2*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)+9*C*arctan((b^2*c)^(1/2
)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^4*c*f^2+12*C*arctan((b^2*c)^(1/2)*x/(-c*(b^
2*x^2-a^2))^(1/2))*a^2*b^2*c*e^2-16*C*x^2*b^2*e*f*(b^2*c)^(1/2)*(-c*(b^2*x^
2-a^2))^(1/2)-12*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b^2*f^2-24*B*(b
^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b^2*e*f-9*C*(b^2*c)^(1/2)*(-c*(b^2*x
^2-a^2))^(1/2)*x*a^2*f^2-12*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x*b^2*
e^2-48*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*b^2*e*f-16*B*(b^2*c)^(1/2)*
(-c*(b^2*x^2-a^2))^(1/2)*a^2*f^2-24*B*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2
)*b^2*e^2-32*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*a^2*e*f)/b^4/(-c*(b^2
*x^2-a^2))^(1/2)/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9133, size = 1065, normalized size = 2.89

$$\left[-\frac{3\left(8\,Ba^2b^2ef+4\left(Ca^2b^2+2\,Ab^4\right)e^2+\left(3\,Ca^4+4\,Aa^2b^2\right)f^2\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/48*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^2 + (3*C*a^4 + 4*A*a
^2*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*
b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 + 16*B*a^2*b*f^2
+ 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)*x^2 + 3*(4*C*b
^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqr
t(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^
2 + (3*C*a^4 + 4*A*a^2*b^2)*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x
 + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 +
16*B*a^2*b*f^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)
*x^2 + 3*(4*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b
*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.29 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=246

$$-\frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)\right.}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] -(C*(e + f*x)^2*(a^2 - b^2*x^2))/(3*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
- ((2*(2*a^2*C*f^2 - b^2*(C*e^2 - 3*f*(B*e + A*f))) - b^2*f*(C*e - 3*B*f)*x
)*(a^2 - b^2*x^2))/(6*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2*e
+ a^2*(C*e + B*f))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c
- b^2*c*x^2]])/(2*b^3*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.400279, antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1610, 1654, 780, 217, 203}

$$-\frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+C\right.}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -(C*(e + f*x)^2*(a^2 - b^2*x^2))/(3*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
- ((2*(2*a^2*C*f^2 - (b^2*(2*C*e^2 - 6*f*(B*e + A*f)))/2) - b^2*f*(C*e - 3*
B*f)*x)*(a^2 - b^2*x^2))/(6*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*
b^2*e + a^2*(C*e + B*f))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[
a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(-c\left(3Ab^2+2a^2C\right)f^2+b^2cf(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{3b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(Ce-3\right.}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(Ce-3\right.}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(Ce-3\right.}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] time = 1.62425, size = 390, normalized size = 1.59

$$-\frac{6\sqrt{a-bx}\sqrt{a+bx}\left(\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}+2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\right)\left(3a^2Cf-2ab(Bf+Ce)+b^2(Af+Be)\right)+Cf\sqrt{a+bx}\left((a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x
]
```

```
[Out] -(6*(3*a^2*C*f + b^2*(B*e + A*f) - 2*a*b*(C*e + B*f))*Sqrt[a - b*x]*Sqrt[a
+ b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a] + 2*Sqrt[a]*ArcSin[Sqrt[a - b*x]/(S
qrt[2]*Sqrt[a])]) + 3*(b*C*e + b*B*f - 3*a*C*f)*Sqrt[a - b*x]*Sqrt[a + b*x]
*(Sqrt[a - b*x]*(4*a + b*x)*Sqrt[1 + (b*x)/a] + 6*a^(3/2)*ArcSin[Sqrt[a - b
*x]/(Sqrt[2]*Sqrt[a])]) + C*f*Sqrt[a + b*x]*((a - b*x)*Sqrt[1 + (b*x)/a]*(2
2*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^(5/2)*Sqrt[a - b*x]*ArcSin[Sqrt[a - b*x
]/(Sqrt[2]*Sqrt[a])]) + 12*(A*b^2 + a*(-(b*B) + a*C))*(b*e - a*f)*Sqrt[a -
b*x]*Sqrt[1 + (b*x)/a]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(6*b^4*Sqrt[c*(
a - b*x)]*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.022, size = 365, normalized size = 1.5

$$\frac{1}{6\,cb^4}\sqrt{bx+a}\sqrt{-c\,(bx-a)}\left(6\,A\arctan\left(\frac{\sqrt{b^2c}x}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)b^4ce+3\,B\arctan\left(\frac{\sqrt{b^2c}x}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)a^2b^2cf+3\,C\arctan\left(\frac{}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/6*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/c*(6*A*arctan((b^2*c)^(1/2)*x/(-c*(b^2
*x^2-a^2))^(1/2))*b^4*c*e+3*B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/
2))*a^2*b^2*c*f+3*C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*a^2*b^
2*c*e-2*C*x^2*b^2*f*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)-3*B*(b^2*c)^(1/2
)*(-c*(b^2*x^2-a^2))^(1/2)*x*b^2*f-3*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/
2)*x*b^2*e-6*A*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*b^2*f-6*B*(b^2*c)^(1/
2)*(-c*(b^2*x^2-a^2))^(1/2)*b^2*e-4*C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2
)*a^2*f)/(-c*(b^2*x^2-a^2))^(1/2)/b^4/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70071, size = 689, normalized size = 2.8

$$\left[-\frac{3\left(Ba^2bf+\left(Ca^2b+2\,Ab^3\right)e\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(2\,Cb^2fx^2+6\,Bb^2e+2\left(2\,Ca^2\right.\right.}{12\,b^4c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2*
sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2 +
 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e + B*b^2*f)*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)
*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a
^2*c)) + (2*C*b^2*f*x^2 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e
+ B*b^2*f)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

Sympy [C] time = 138.497, size = 736, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*A*a*f*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2,
0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**2*sqrt(c)) - A*a*f*meijerg(((-1,
 -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), a**2*ex
p_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**2*sqrt(c)) - I*A*e*meijerg(((
1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**2/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(c)) + A*e*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), (
)), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4
*pi**(3/2)*b*sqrt(c)) - I*B*a**2*f*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1
)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**3
*sqrt(c)) + B*a**2*f*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4,
 -3/4), (-3/2, -1, -1, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/
2)*b**3*sqrt(c)) - I*B*a*e*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -
1/4, 0, 1/4, 1/2, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**2*sqrt(c)) - B
*a*e*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2,
-1/2, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**2*sqrt(c))
- I*C*a**3*f*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -
3/4, -1/2, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**4*sqrt(c)) - C*a**3*f
*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/
2, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**4*sqrt(c)) - I
*C*a**2*e*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/
4, 0, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt(c)) + C*a**2*e*meij
erg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)
), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt(c))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.30 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=177

$$\frac{\left(a^2C+2Ab^2\right)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] -((B*(a^2 - b^2*x^2))/(b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - (C*x*(a^2 -
b^2*x^2))/(2*b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2 + a^2*C)*Sqrt
[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b^3*S
qrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.124376, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {901, 1815, 641, 217, 203}

$$\frac{\left(a^2C+2Ab^2\right)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -((B*(a^2 - b^2*x^2))/(b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - (C*x*(a^2 -
b^2*x^2))/(2*b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2 + a^2*C)*Sqrt
[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b^3*S
qrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-c\left(2Ab^2+a^2C\right)-2b^2Bcx}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2}}}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\right)\operatorname{Subst}\left(\int\right.}{2b^2\sqrt{a+bx}\sqrt{ac-}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b}{\sqrt{a^2c}}\right.}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] time = 0.414243, size = 169, normalized size = 0.95

$$-\frac{\sqrt{a-bx}\left(\sqrt{\frac{bx}{a}+1}\left(4\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)\left(a(aC-bB)+Ab^2\right)+b\sqrt{a-bx}\sqrt{a+bx}(2B+Cx)\right)-2\sqrt{a}\sqrt{a+bx}(aC-2bB)\sin\right.}{2b^3\sqrt{\frac{bx}{a}+1}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -(Sqrt[a - b*x]*(-2*Sqrt[a]*(-2*b*B + a*C)*Sqrt[a + b*x]*ArcSin[Sqrt[a - b*
x]/(Sqrt[2]*Sqrt[a])] + Sqrt[1 + (b*x)/a]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(2
*B + C*x) + 4*(A*b^2 + a*(-(b*B) + a*C))*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]
])))/(2*b^3*Sqrt[c*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.018, size = 180, normalized size = 1.

$$\frac{1}{2\,b^2c}\sqrt{bx+a}\sqrt{-c\,(bx-a)}\left(2\,A\arctan\left(\frac{\sqrt{b^2c}x}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)b^2c+C\arctan\left(x\sqrt{b^2c}\frac{1}{\sqrt{-c\left(b^2x^2-a^2\right)}}\right)a^2c-C\sqrt{b^2c}\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/b^2*(2*A*arctan((b^2*c)^(1/2)*x/(-c*(b
^2*x^2-a^2))^(1/2))*b^2*c+C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2)
)*a^2*c-C*(b^2*c)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*x-2*B*(-c*(b^2*x^2-a^2))^(
1/2)*(b^2*c)^(1/2))/(-c*(b^2*x^2-a^2))^(1/2)/c/(b^2*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6435, size = 460, normalized size = 2.6

$$\left[-\frac{\left(Ca^2+2\,Ab^2\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+ac}\sqrt{bx+a}}{4\,b^3c},-\frac{\left(Ca^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sq
rt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

Sympy [C] time = 25.8753, size = 338, normalized size = 1.91

$$-\frac{iAG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{AG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}-\frac{iBaG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*A*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + A*meijerg(((-1/2, -1/4, 0, 1/
4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b
**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) - I*B*a*meijerg(((-1/4, 1/4), (0, 0, 1/2
, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b
**2*sqrt(c)) - B*a*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4
), (-1, -1/2, -1/2, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*
b**2*sqrt(c)) - I*C*a**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1,
-3/4, -1/2, -1/4, 0, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt(c))
+ C*a**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/
2, -1, -1, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt
(c))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac
")

[Out] Timed out
```

3.31 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C\left(a^2-\right.}{b^2f\sqrt{a+bx}}$$

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.464124, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 844, 217, 203, 725, 204}

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C\left(a^2-\right.}{b^2f\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx &= \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\left(\left(Ce^2-Be\right.\right.}{} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b}}\right.}{f^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{\left(Ce^2-Be\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.729006, size = 225, normalized size = 0.81

$$
\frac{\sqrt{a-bx}\left(\frac{2\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{af-be}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)(aCf-bBf+bCe)}{b^2}+\frac{Cf\sqrt{a+bx}\left(-\sqrt{a-bx}-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{b^2}\right)}{f^2\sqrt{c(a-bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] (Sqrt[a - b*x]*((C*f*Sqrt[a + b*x]*(-Sqrt[a - b*x] - (2*Sqrt[a]*ArcSin[Sqrt
[a - b*x]/(Sqrt[2]*Sqrt[a])])/Sqrt[1 + (b*x)/a]))/b^2 + (2*(b*C*e - b*B*f +
 a*C*f)*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/b^2 + (2*(C*e^2 + f*(-(B*e) +
A*f))*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a
+ b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[-(b*e) + a*f])))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0., size = 503, normalized size = 1.8

$$\frac{1}{b^2f^3c}\left(-A\ln\left(2\frac{1}{fx+e}\left(b^2cex+a^2cf+\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{-c\left(b^2x^2-a^2\right)}f\right)\right)b^2cf^2\sqrt{b^2c}+B\ln\left(2\frac{1}{fx+e}\left(b^2cex+a^2cf\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (-A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^
2))^(1/2)*f)/(f*x+e))*b^2*c*f^2*(b^2*c)^(1/2)+B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e*f
*(b^2*c)^(1/2)+B*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*f^2
*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*(b^2*c)^(1/2
)-C*arctan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*e*f*(c*(a^2*f^2-
b^2*e^2)/f^2)^(1/2)-C*f^2*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2))*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)/b^2/(c*(a^2*f^2-b^2
*e^2)/f^2)^(1/2)/(b^2*c)^(1/2)/f^3/c/(-c*(b^2*x^2-a^2))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.32 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} \, dx$

Optimal. Leaf size=322

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^{3/2}}$$

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rubi [A] time = 0.530435, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1651, 844, 217, 203, 725, 204}

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{c\left(Ab^2e+a^2(Ce-Bf)\right)+cC\left(\frac{b^2 \cdots}{\cdots}\right. \cdots}{(e+fx)\sqrt{a^2c-b^2cx^2} \cdots}}{c\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac- \cdots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{\cdots}{\sqrt{a^2c \cdots}}}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-b \cdots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\mathrm{Subst} \cdots}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+ \cdots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \cdots$$

Mathematica [A] time = 0.968873, size = 309, normalized size = 0.96

$$\frac{\frac{2b^2e\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(af-be)^{3/2}} + \frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)(af-be)(af+be)} - \frac{2\sqrt{a-bx}(2Ce-Bf)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{af-be}} - \frac{2C\sqrt{a-bx}\tan^{-1}\left(\cdots\right)}{b} \cdots}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)) - (2*C*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x
]])/b - (2*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a -
b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[-(b*e)
+ a*f]) + (2*b^2*e*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*ArcTan[(Sqrt[-(
b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/((-(b*e) -
a*f)^(3/2)*(-(b*e) + a*f)^(3/2)))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0., size = 1200, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2
))^(1/2)*f)/(f*x+e))*x*b^2*c*e*f^3*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*
c*f^4*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*c*e*f^3*(b^2*c)^(1/2)-C*ln(
2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/
2)*f)/(f*x+e))*x*b^2*c*e^3*f*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(-c*(b^
2*x^2-a^2))^(1/2))*x*a^2*c*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^
2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*x*b^2*c*e^2*f^2*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(
b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^
(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^
2-a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/
(-c*(b^2*x^2-a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arc
tan((b^2*c)^(1/2)*x/(-c*(b^2*x^2-a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e
^2)/f^2)^(1/2)-A*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2
*e^2)/f^2)^(1/2)+B*e*f^3*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a^2*f^2
-b^2*e^2)/f^2)^(1/2)-C*e^2*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(b^2*c)^(1/2)*(c*(a
^2*f^2-b^2*e^2)/f^2)^(1/2))/c*(-c*(b*x-a))^(1/2)*(b*x+a)^(1/2)/(-c*(b^2*x^2
-a^2))^(1/2)/(a*f+b*e)/(a*f-b*e)/(f*x+e)/(b^2*c)^(1/2)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{(a^2-b^2x^2)\left(2a^2f^2(2Ce-Bf)-b^2e\left(f(Be-3Af)+Ce^2\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}}{}$$

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*e*
(C*e^2 + f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((A*(2*b^4*e^2 + a^2*b^2*f^2) + a^
2*(2*a^2*C*f^2 + b^2*e*(C*e - 3*B*f)))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt
[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/
(2*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.58763, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1610, 1651, 807, 725, 204}

$$\frac{(a^2-b^2x^2)\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{2c\left(Ab^2e+a^2(Ce-Bf)\right)-c\ldots}{(e+fx)^2\ldots}}{2c\left(b^2e^2-a^2f^2\right)\sqrt{a+\ldots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3+ef(\ldots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\ldots}$$

Mathematica [A] time = 1.83553, size = 492, normalized size = 1.36

$$\frac{\frac{b^2\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\left(2(e+fx)\left(a^2f^2+2b^2e^2\right)\tan^{-1}\left(\frac{\sqrt{a-bx}\sqrt{af-be}}{\sqrt{a+bx}\sqrt{-af-be}}\right)+3ef\sqrt{a-bx}\sqrt{a+bx}\sqrt{-af-be}\sqrt{af-be}\right)}{(e+fx)(-af-be)^{5/2}(af-be)^{5/2}}+\frac{2f(bx-a)\sqrt{a+bx}(Bf-2Ce)}{(e+fx)\left(a^2f^2-b^2e^2\right)}+\frac{f(bx-a)\sqrt{a+\ldots}}{(e+fx)^2\ldots}}{2f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)^2) + (2*f*(-2*C*e + B*f)*(-a + b*x)*Sqrt[a + b*x])/((-(
b^2*e^2) + a^2*f^2)*(e + f*x)) + (4*C*Sqrt[a - b*x]*ArcTan[(Sqrt[-(b*e) + a
*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/(Sqrt[-(b*e) - a*f]
```

```
*Sqrt[-(b*e) + a*f]) - (4*b^2*e*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTan[(Sqrt[-(
b*e) + a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) -
a*f)^(3/2)*(-(b*e) + a*f)^(3/2)) + (b^2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a -
 b*x]*(3*e*f*Sqrt[-(b*e) - a*f]*Sqrt[-(b*e) + a*f]*Sqrt[a - b*x]*Sqrt[a + b
*x] + 2*(2*b^2*e^2 + a^2*f^2)*(e + f*x)*ArcTan[(Sqrt[-(b*e) + a*f]*Sqrt[a -
 b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]))/((-(b*e) - a*f)^(5/2)*(-(b*e)
+ a*f)^(5/2)*(e + f*x)))/(2*f^2*Sqrt[c*(a - b*x)])
```

Maple [B] time = 0., size = 1848, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2*(C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^
2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*e^2*f^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f
+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2
*b^2*c*e*f^3-6*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-
c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^2*b^2*c*e^2*f^2+2*C*ln(2*(b^2*c*e*x+
a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e)
)*x*a^2*b^2*c*e^3*f+A*a^2*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c
*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*e*f^3+2*A*ln(2*(b^2*c*e*x+a
^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))
*b^4*c*e^4+4*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*
(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*a^4*c*e*f^3+A*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*
c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(
b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^3*f-3*A*x*b^2*e*f^3*(-c*(b^2*x^
2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*x*b^2*e^2*f^2*(-c*(b^2*x^2-
a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*C*x*a^2*e*f^3*(-c*(b^2*x^2-a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+C*x*b^2*e^3*f*(-c*(b^2*x^2-a^2))^
(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2
-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*f^4
+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a
^2))^(1/2)*f)/(f*x+e))*x^2*b^4*c*e^2*f^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^
2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*x*b^4*c*e^3*
f+2*B*x*a^2*f^4*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*
A*b^2*e^2*f^2*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*a^
2*e*f^3*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^2*e^
3*f*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^2*e^2*f^
2*(-c*(b^2*x^2-a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*ln(2*(b^2*c*
e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*
x+e))*x^2*a^4*c*f^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a
^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-c*(b^2*x^2-a^2))^(1/2)*f)/(f*x+e))
*a^2*b^2*c*e^4)/c*(-c*(b*x-a))^(1/2)*(b*x+a)^(1/2)/(-c*(b^2*x^2-a^2))^(1/2)
/(a*f+b*e)/(a*f-b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)/f
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 10.6782, size = 2238, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*C*a^4*sqrt(-c)*c^2*f^2 + A*a^2*b^2*sqrt(-c)*c^2*f^2 - 3*B*a^2*b^2*sqrt(
-c)*c^2*f*e + C*a^2*b^2*sqrt(-c)*c^2*e^2 + 2*A*b^4*sqrt(-c)*c^2*e^2)*arctan
(1/2*(2*b*c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*
c)*c))^2*f)/(sqrt(a^2*f^2 - b^2*e^2)*c^2))/((a^4*f^4*abs(c) - 2*a^2*b^2*f^2
*abs(c)*e^2 + b^4*abs(c)*e^4)*sqrt(a^2*f^2 - b^2*e^2)*c^2) + 2*(16*B*a^6*b*
sqrt(-c)*c^8*f^5 - 32*C*a^6*b*sqrt(-c)*c^8*f^4*e - 24*A*a^4*b^3*sqrt(-c)*c^
8*f^4*e + 4*A*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2*sqrt(-c)*c^6*f^5 + 8*B*a^4*b^3*sqrt(-c)*c^8*f^3*e^2 + 20*B*a^4
*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt
(-c)*c^6*f^4*e + 4*B*a^4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b
*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^5 + 8*C*a^4*b^3*sqrt(-c)*c^8*f^2*e^3 - 44*
C*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2
*sqrt(-c)*c^6*f^3*e^2 - 40*A*a^2*b^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*
a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^3*e^2 - 8*C*a^4*b*(sqrt(-b*c*x +
```

```
 a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^4*e - 6*
A*a^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4
*sqrt(-c)*c^4*f^4*e - A*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2
 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^5 + 16*B*a^2*b^4*(sqrt(-b*c*x + a*c)*
sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^2*e^3 + 10*B*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^3*e^2 + 3*B*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^
2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^4*e + 8*C*a^2*b^4*(sqrt(-b*c*x + a*c
)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f*e^4 - 14*C*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^2*e^3 - 12*A*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 +
 (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^2*e^3 - 5*C*a^2*b^2*(sqrt(-b*c*x + a*c)
*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^3*e^2 - 2*A*b
^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-
c)*c^2*f^3*e^2 + 4*B*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^4*sqrt(-c)*c^4*f*e^4 + 4*C*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) -
 sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*e^5 + 2*C*b^4*(sqrt(-b*c*x
 + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f*e^4)/(
(a^4*f^6*abs(c) - 2*a^2*b^2*f^4*abs(c)*e^2 + b^4*f^2*abs(c)*e^4)*(4*a^2*c^4
*f + 4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*
c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*f
)^2)
```

3.34 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}+\frac{cx^2\sqrt{dx-1}\sqrt{dx+1}}{3d^2}$$

```
[Out] (c*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*d^2) + (Sqrt[-1 + d*x]*Sqrt[1 + d*x
]*(2*(2*c + 3*a*d^2) + 3*b*d^2*x))/(6*d^4) + (b*ArcCosh[d*x])/(2*d^3)
```

Rubi [A] time = 0.146105, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1610, 1809, 780, 217, 206}

$$-\frac{\left(1-d^2x^2\right)\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*(1 - d^2*x^2))/(3*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c + 3*
a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (
b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*
x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(2c+3ad^2+3bd^2x\right)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d\ldots}\right.}{2d^2\sqrt{-1+dx}\sqrt{1\ldots}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2\ldots}}\right.}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.344116, size = 149, normalized size = 1.71

$$\frac{\sqrt{-(dx-1)^2}\sqrt{dx+1}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)+6\sqrt{dx-1}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)(d(2ad-b)+2c)-12\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\ldots}\right.}{6d^4\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))
+ 6*(2*c + d*(-b + 2*a*d))*Sqrt[-1 + d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] - 1
2*(c + d*(-b + a*d))*Sqrt[1 - d*x]*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*
d^4*Sqrt[1 - d*x])
```

Maple [C] time = 0., size = 137, normalized size = 1.6

$$\frac{\text{csgn}(d)}{6d^4}\sqrt{dx-1}\sqrt{dx+1}\left(2\,\text{csgn}(d)\,x^2cd^2\sqrt{d^2x^2-1}+3\,\text{csgn}(d)\,\sqrt{d^2x^2-1}xbd^2+6\,\text{csgn}(d)\,\sqrt{d^2x^2-1}ad^2+4\,\text{csgn}(d)\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(d^2*x^2-1)^(1/2)+3*cs
gn(d)*(d^2*x^2-1)^(1/2)*x*b*d^2+6*csgn(d)*(d^2*x^2-1)^(1/2)*a*d^2+4*csgn(d)
```

```
*(d^2*x^2-1)^(1/2)*c+3*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

Maxima [A] time = 2.13899, size = 147, normalized size = 1.69

$$\frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}+\frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(d^2))/(sqrt(d
^2)*d^2) + 2/3*sqrt(d^2*x^2 - 1)*c/d^4
```

Fricas [A] time = 1.62428, size = 176, normalized size = 2.02

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [C] time = 44.2604, size = 308, normalized size = 3.54

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d**2) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1
, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*mei
jerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0
)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + c*meijerg(((-5/4, -
3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x*
*2))/(4*pi**(3/2)*d**4) + I*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
```

```
(3/2)*d**4)
```

Giac [A] time = 2.21004, size = 130, normalized size = 1.49

$$-\frac{6\,bd^{10}\log\left(\left|-\sqrt{dx+1}+\sqrt{dx-1}\right|\right)-\left(6\,ad^{11}-3\,bd^{10}+6\,cd^{9}+\left(2\,(dx+1)cd^{9}+3\,bd^{10}-4\,cd^{9}\right)(dx+1)\right)\sqrt{dx+1}\sqrt{dx-1}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3840*(6*b*d^10*log(abs(-sqrt(d*x + 1) + sqrt(d*x - 1))) - (6*a*d^11 - 3*
b*d^10 + 6*c*d^9 + (2*(d*x + 1)*c*d^9 + 3*b*d^10 - 4*c*d^9)*(d*x + 1))*sqrt
(d*x + 1)*sqrt(d*x - 1))/d
```

3.35 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2ad^2+c)\cosh^{-1}(dx)}{2d^3}+\frac{\sqrt{dx-1}\sqrt{dx+1}(2b+cx)}{2d^2}$$

```
[Out] ((2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*d^2) + ((c + 2*a*d^2)*ArcCosh
[d*x])/(2*d^3)
```

Rubi [B] time = 0.0706656, antiderivative size = 135, normalized size of antiderivative = 2.6, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {901, 1815, 641, 217, 206}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,d\right.}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.213604, size = 126, normalized size = 2.42

$$\frac{4\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)+d\sqrt{-(dx-1)^2}\sqrt{dx+1}(2b+cx)+2\sqrt{dx-1}(2bd-c)\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)}{2d^3\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x] + 2*(-c + 2*b*d)*Sqrt[-1 +
 d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + 4*(c + d*(-b + a*d))*Sqrt[1 - d*x]*Ar
cTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(2*d^3*Sqrt[1 - d*x])
```

Maple [C] time = 0., size = 120, normalized size = 2.3

$$\frac{\text{csgn}(d)}{2\,d^3}\sqrt{dx-1}\sqrt{dx+1}\left(\text{csgn}(d)\,d\sqrt{d^2x^2-1}xc+2\,\text{csgn}(d)\,d\sqrt{d^2x^2-1}b+2\,\ln\left(\left(\text{csgn}(d)\,\sqrt{d^2x^2-1}+dx\right)\text{csgn}(d)\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(csgn(d)*d*(d^2*x^2-1)^(1/2)*x*c+2*csgn(d)*
d*(d^2*x^2-1)^(1/2)*b+2*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*a*d^2+l
n((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*c)*csgn(d)/d^3/(d^2*x^2-1)^(1/2)
```

Maxima [B] time = 2.83955, size = 142, normalized size = 2.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}} + \frac{\sqrt{d^2x^2-1}cx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}b}{d^2} + \frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(d^2))/sqrt(d^2) + 1/2*sqrt(d^2*x^2
 - 1)*c*x/d^2 + sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^
2 - 1)*sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.63294, size = 150, normalized size = 2.88

$$\frac{(cdx + 2\,bd)\sqrt{dx + 1}\sqrt{dx - 1} - \left(2\,ad^2 + c\right)\log\left(-dx + \sqrt{dx + 1}\sqrt{dx - 1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [C] time = 21.3621, size = 277, normalized size = 5.33

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1
/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*meijerg(((-1, -3/4, -1
/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**2) + c*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0
, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3
) - I*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2
, -1, -1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 2.51824, size = 104, normalized size = 2.

$$\frac{\left((dx + 1)cd^4 + 2\,bd^5 - cd^4\right)\sqrt{dx + 1}\sqrt{dx - 1} - 2\left(2\,ad^6 + cd^4\right)\log\left(\left|-\sqrt{dx + 1} + \sqrt{dx - 1}\right|\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(((d*x + 1)*c*d^4 + 2*b*d^5 - c*d^4)*sqrt(d*x + 1)*sqrt(d*x - 1) - 2*
(2*a*d^6 + c*d^4)*log(abs(-sqrt(d*x + 1) + sqrt(d*x - 1))))/d
```

3.36 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$a\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\cosh^{-1}(dx)}{d}+\frac{c\sqrt{dx-1}\sqrt{dx+1}}{d^2}$$

```
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + (b*ArcCosh[d*x])/d + a*ArcTan[Sqrt[-
1 + d*x]*Sqrt[1 + d*x]]
```

Rubi [B] time = 0.184666, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1610, 1809, 844, 217, 206, 266, 63, 205}

$$\frac{a\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}-\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{S}\cdots}{\sqrt{\cdots}}$$
$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{\cdots}{\frac{1}{d^2}\cdots}\right.}{d^2\sqrt{-1+dx}\cdots}$$
$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{d\cdots}{\sqrt{-1+\cdots}}\right.}{d\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.406817, size = 128, normalized size = 2.33

$$\frac{\frac{ad^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)+cd^2x^2-2c\sqrt{1-d^2x^2}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)-c}{\sqrt{dx-1}\sqrt{dx+1}}-2(c-bd)\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-c + c*d^2*x^2 - 2*c*Sqrt[1 - d^2*x^2]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + a*
d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 +
 d*x]) - 2*(c - b*d)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d^2
```

Maple [C] time = 0., size = 95, normalized size = 1.7

$$\frac{\operatorname{csgn}(d)}{d^2}\left(-\operatorname{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)ad^2+\operatorname{csgn}(d)\sqrt{d^2x^2-1}c+\ln\left(\left(\operatorname{csgn}(d)\sqrt{(dx+1)(dx-1)}+dx\right)\operatorname{csgn}(d)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+csgn(d)*(d^2*x^2-1)^(1/2)*c+ln(
(csgn(d)*((d*x+1)*(d*x-1))^(1/2)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Maxima [A] time = 2.30598, size = 86, normalized size = 1.56

$$-a\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}}+\frac{\sqrt{d^2x^2-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(sqrt(d^2)*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(
d^2))/sqrt(d^2) + sqrt(d^2*x^2 - 1)*c/d^2
```

Fricas [A] time = 1.55962, size = 184, normalized size = 3.35

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] time = 26.9759, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_6^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A] time = 2.13435, size = 96, normalized size = 1.75

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

3.37 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{dx-1}\sqrt{dx+1}}{x}+b\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{c\cosh^{-1}(dx)}{d}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + (c*ArcCosh[d*x])/d + b*ArcTan[Sqrt[-1
+ d*x]*Sqrt[1 + d*x]]
```

Rubi [B] time = 0.179683, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1610, 1807, 844, 217, 206, 266, 63, 205}

$$-\frac{a\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\mathrm{S}\ldots}{\sqrt{\ldots}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{\ldots}{\frac{1}{d^2}\ldots}\right.}{d^2\sqrt{-1+dx}\ldots}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{d\ldots}{\sqrt{-1+\ldots}}\right.}{d\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.16739, size = 89, normalized size = 1.62

$$\frac{a\left(d^2x^2-1\right)+bx\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{x\sqrt{dx-1}\sqrt{dx+1}} + \frac{2c\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*(-1 + d^2*x^2) + b*x*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(x*S
qrt[-1 + d*x]*Sqrt[1 + d*x]) + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] time = 0., size = 96, normalized size = 1.8

$$\frac{\mathrm{csgn}\,(d)}{dx}\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}\,(d)\,dxb+\mathrm{csgn}\,(d)\,d\sqrt{d^2x^2-1}a+\ln\left(\left(\mathrm{csgn}\,(d)\,\sqrt{d^2x^2-1}+dx\right)\mathrm{csgn}\,(d)\right)xc\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-arctan(1/(d^2*x^2-1)^(1/2))*csgn(d)*d*x*b+csgn(d)*d*(d^2*x^2-1)^(1/2)*a+l
n((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*x*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/d/x
```

Maxima [A] time = 3.56079, size = 86, normalized size = 1.56

$$-b\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\sqrt{d^2}\right)}{\sqrt{d^2}}+\frac{\sqrt{d^2x^2-1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(sqrt(d^2)*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*sqrt(
d^2))/sqrt(d^2) + sqrt(d^2*x^2 - 1)*a/x
```

Fricas [A] time = 1.1004, size = 203, normalized size = 3.69

$$\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] time = 28.1729, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\\frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Giac [A] time = 2.62059, size = 112, normalized size = 2.04

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)-\frac{8\,ad^{2}}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

3.38 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{2x^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{x}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
/x + ((2*c + a*d^2)*ArcTan[Sqrt[-1 + d*x]*Sqrt[1 + d*x]])/2
```

Rubi [A] time = 0.191248, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1610, 1807, 807, 266, 63, 205}

$$\frac{\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*ArcTa
n[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+\left(2c+ad^2\right)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,d\ldots}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{\ldots}{x\sqrt{-\ldots}}\ldots\right.}{4\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\ldots}\ldots\right.}{2d^2\sqrt{-1+dx}\sqrt{1+\ldots}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d\ldots}\right.}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.116759, size = 82, normalized size = 0.99

$$\frac{\left(d^2x^2-1\right)(a+2bx)+x^2\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*(-1 + d^2*x^2) + (2*c + a*d^2)*x^2*Sqrt[-1 + d^2*x^2]*ArcTan[S
qrt[-1 + d^2*x^2]])/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Maple [C] time = 0., size = 103, normalized size = 1.2

$$-\frac{(\text{csgn}(d))^2}{2x^2}\sqrt{dx-1}\sqrt{dx+1}\left(\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^2ad^2+2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^2c-2\sqrt{d^2x^2-1}xb-\sqrt{d^2x^2-1}a\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*x^2
*a*d^2+2*arctan(1/(d^2*x^2-1)^(1/2))*x^2*c-2*(d^2*x^2-1)^(1/2)*x*b-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

Maxima [A] time = 4.01953, size = 88, normalized size = 1.06

$$-\frac{1}{2}\,ad^{2}\arcsin\left(\frac{1}{\sqrt{d^{2}}|x|}\right)-c\arcsin\left(\frac{1}{\sqrt{d^{2}}|x|}\right)+\frac{\sqrt{d^{2}x^{2}-1}\,b}{x}+\frac{\sqrt{d^{2}x^{2}-1}\,a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(sqrt(d^2)*abs(x))) - c*arcsin(1/(sqrt(d^2)*abs(x))) +
sqrt(d^2*x^2 - 1)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Fricas [A] time = 1.17296, size = 173, normalized size = 2.08

$$\frac{2\,bdx^{2}+2\left(ad^{2}+2\,c\right)x^{2}\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\left(2\,bx+a\right)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [C] time = 33.6278, size = 212, normalized size = 2.55

$$-\frac{ad^{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{iad^{2}G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibd\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4
, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)) - c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5
/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*c*meijerg(((0, 1/4, 1/2,
```

```
3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x*
*2))/(4*pi**(3/2))
```

Giac [B] time = 1.98699, size = 196, normalized size = 2.36

$$-\frac{\left(ad^3+2\,cd\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

3.39 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2ad^2+3c\right)}{3x}+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{3x^3}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{2x^2}$$

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*x^3) + (b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
/(2*x^2) + ((3*c + 2*a*d^2)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(3*x) + (b*d^2*Ar
cTan[Sqrt[-1 + d*x]*Sqrt[1 + d*x]])/2
```

Rubi [A] time = 0.217163, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1610, 1807, 835, 807, 266, 63, 205}

$$-\frac{\left(1-d^2x^2\right)\left(2ad^2+3c\right)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(3*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(3*
x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1
+ d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+\left(3c+2ad^2\right)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2\left(3c+2ad^2\right)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}\,dx}{6\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+\right.}{2\sqrt{-}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{bd^2\sqrt{-1+}}{2\sqrt{}}$$

Mathematica [A] time = 0.116358, size = 94, normalized size = 0.81

$$\frac{\left(d^2x^2-1\right)\left(a\left(4d^2x^2+2\right)+3x(b+2cx)\right)+3bd^2x^3\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{6x^3\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-1 + d^2*x^2)*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)) + 3*b*d^2*x^3*Sqrt[-1
 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(6*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]
)
```

Maple [C] time = 0., size = 123, normalized size = 1.1

$$-\frac{(\mathrm{csgn}(d))^2}{6x^3}\sqrt{dx-1}\sqrt{dx+1}\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)x^3bd^2-4\sqrt{d^2x^2-1}x^2ad^2-6\sqrt{d^2x^2-1}x^2c-3\sqrt{d^2x^2-1}xb-2\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*x
^3*b*d^2-4*(d^2*x^2-1)^(1/2)*x^2*a*d^2-6*(d^2*x^2-1)^(1/2)*x^2*c-3*(d^2*x^2
-1)^(1/2)*x*b-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

Maxima [A] time = 3.64281, size = 119, normalized size = 1.03

$$-\frac{1}{2}bd^2\arcsin\left(\frac{1}{\sqrt{d^2}|x|}\right)+\frac{2\sqrt{d^2x^2-1}ad^2}{3x}+\frac{\sqrt{d^2x^2-1}c}{x}+\frac{\sqrt{d^2x^2-1}b}{2x^2}+\frac{\sqrt{d^2x^2-1}a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(sqrt(d^2)*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + s
qrt(d^2*x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*
a/x^3
```

Fricas [A] time = 1.0307, size = 216, normalized size = 1.86

$$\frac{6bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\left(2ad^3+3cd\right)x^3+\left(2\left(2ad^2+3c\right)x^2+3bx+2a\right)\sqrt{dx+1}\sqrt{dx-1}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [C] time = 58.436, size = 219, normalized size = 1.89

$$-\frac{ad^{3}G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{iad^{3}G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bd^{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2,
9/4, 5/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*d**2*meijerg(((1, 5/4,
 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)) - c*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1,
 5/4, 3/2, 7/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*c*d*meijerg(((1
/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*
pi)/(d**2*x**2))/(4*pi**(3/2))
```

Giac [B] time = 2.36273, size = 266, normalized size = 2.29

$$-\frac{3\,bd^{3}\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)+\frac{2\left(3\,bd^{3}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{8}-96\,ad^{4}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}-96\,cd^{2}\left(\sqrt{dx+1}-\cdots\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}+4\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

3.40 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{x-1}\sqrt{x+1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\sqrt{x-1}\sqrt{x+1}\left(-de^2(3a+4c)+bd^2e+2be^3+cd^3\right)}{2e\left(d^2-e^2\right)^2(d+ex)}+\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}\sqrt{d+e}}{\sqrt{x-1}\sqrt{d-e}}\right)\left(d^2(2a+c)\right.}{(d-e)^{5/2}(d+}$$

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x]*Sqrt[1 + x])/(2*e*(d^2 - e^2)*(d + e
*x)^2) + ((c*d^3 + b*d^2*e - (3*a + 4*c)*d*e^2 + 2*b*e^3)*Sqrt[-1 + x]*Sqrt
[1 + x])/(2*e*(d^2 - e^2)^2*(d + e*x)) + (((2*a + c)*d^2 - 3*b*d*e + (a + 2
*c)*e^2)*ArcTanh[(Sqrt[d + e]*Sqrt[1 + x])/(Sqrt[d - e]*Sqrt[-1 + x])])/((d
 - e)^(5/2)*(d + e)^(5/2))
```

Rubi [A] time = 0.328291, antiderivative size = 242, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1610, 1651, 807, 725, 206}

$$-\frac{\left(1-x^2\right)\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\sqrt{x-1}\sqrt{x+1}\left(d^2-e^2\right)^2(d+ex)}+\frac{\left(1-x^2\right)\left(ae^2-bde+cd^2\right)}{2e\sqrt{x-1}\sqrt{x+1}\left(d^2-e^2\right)(d+ex)^2}-\frac{\sqrt{x^2-1}\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)}{2\sqrt{x-1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(1 - x^2))/(2*e*(d^2 - e^2)*Sqrt[-1 + x]*Sqrt[1 +
x]*(d + e*x)^2) - ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*(1 -
 x^2))/(2*e*(d^2 - e^2)^2*Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)) - ((3*b*d*e -
 a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Sqrt[-1 + x^2]*ArcTanh[(e + d*x)/(Sqrt[
d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2)*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx &= \frac{\sqrt{-1+x^2}\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}} \\
&= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{\sqrt{-1+x^2}\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}} \\
&= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)(1-}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)} \\
&= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)(1-}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)} \\
&= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)(1-}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)}
\end{aligned}
$$

Mathematica [A] time = 0.8129, size = 336, normalized size = 1.69

$$
\frac{\frac{\left(2\left(2d^2+e^2\right)(d+ex)\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{x+1}}\sqrt{e-d}}{\sqrt{d+e}}\right)-3de\sqrt{x-1}\sqrt{x+1}\sqrt{e-d}\sqrt{d+e}\right)\left(e(ae-bd)+cd^2\right)}{(e-d)^{5/2}(d+e)^{5/2}(d+ex)} - \frac{e\sqrt{x-1}\sqrt{x+1}\left(e(ae-bd)+cd^2\right)}{(d-e)(d+e)(d+ex)^2} + \frac{2e\sqrt{x-1}\sqrt{x+1}(2cd-be)}{(d-e)(d+e)(d+ex)} + \frac{4d(2cd-be)}{(e-}}{2e^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] (-((e*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[-1 + x]*Sqrt[1 + x])/((d - e)*(d + e)
*(d + e*x)^2)) + (2*e*(2*c*d - b*e)*Sqrt[-1 + x]*Sqrt[1 + x])/((d - e)*(d +
 e)*(d + e*x)) + (4*c*ArcTan[(Sqrt[-d + e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[d +
 e]])/(Sqrt[-d + e]*Sqrt[d + e]) + (4*d*(2*c*d - b*e)*ArcTan[(Sqrt[-d + e]*
Sqrt[(-1 + x)/(1 + x)])/Sqrt[d + e]])/((-d + e)^(3/2)*(d + e)^(3/2)) + ((c*
d^2 + e*(-(b*d) + a*e))*(-3*d*e*Sqrt[-d + e]*Sqrt[d + e]*Sqrt[-1 + x]*Sqrt[
1 + x] + 2*(2*d^2 + e^2)*(d + e*x)*ArcTan[(Sqrt[-d + e]*Sqrt[(-1 + x)/(1 +
x)])/Sqrt[d + e]]))/((-d + e)^(5/2)*(d + e)^(5/2)*(d + e*x)))/(2*e^2)
```

Maple [B] time = 0.052, size = 1095, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] -1/2*(-2*b*d^3*e*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)-b*d*e^3*((d^2-e^2)/e^2
)^(1/2)*(x^2-1)^(1/2)+3*c*d^2*e^2*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+2*ln(
-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*a*d^2*e^2-3*
ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*b*d*e^3+l
n(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*c*d^2*e^2+
4*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*a*d^3*e+2
*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*a*d*e^3-6*
ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*b*d^2*e^2+4
*a*d^2*e^2*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)-x*b*d^2*e^2*((d^2-e^2)/e^2)^
(1/2)*(x^2-1)^(1/2)+ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e
*x+d))*x^2*a*e^4+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*
x+d))*x^2*c*e^4+ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d
))*a*d^2*e^2-3*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d)
)*b*d^3*e+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*c
*d^2*e^2-a*e^4*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+2*ln(-2*(-((d^2-e^2)/e^2
)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*a*d^4+ln(-2*(-((d^2-e^2)/e^2)^(1/2)
*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*c*d^4-x*c*d^3*e*((d^2-e^2)/e^2)^(1/2)*(x^2
-1)^(1/2)+4*x*c*d*e^3*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+3*x*a*d*e^3*((d^2
-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2
)*e+d*x+e)/(e*x+d))*x*c*d^3*e+4*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)
*e+d*x+e)/(e*x+d))*x*c*d*e^3-2*x*b*e^4*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2))
*(1+x)^(1/2)*(-1+x)^(1/2)/(x^2-1)^(1/2)/(d-e)/(d+e)/(d^2-e^2)/(e*x+d)^2/((d
^2-e^2)/e^2)^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.24824, size = 2485, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*sqrt(x + 1)*sqrt(x - 1) + sqr
t(d^2 - e^2)*(d*x + e))/(e*x + d)) + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 - b
*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3 -
 (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x +
 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4
+ (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^
2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e
^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*
d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a
 + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a +
c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e
^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*
e^5)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*e*sqrt(x + 1)*sqrt(x - 1
) - sqrt(-d^2 + e^2)*(e*x + d))/(d^2 - e^2)) + (2*b*d^5*e^2 - (4*a + 3*c)*d
^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b
*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x
)*sqrt(x + 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 +
b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^
4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3
 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.45873, size = 817, normalized size = 4.11

$$-\frac{\left(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2\right)\arctan\left(\frac{\left(\sqrt{x+1}-\sqrt{x-1}\right)^2e+2\,d}{2\,\sqrt{-d^2+e^2}}\right)}{\left(d^4-2\,d^2e^2+e^4\right)\sqrt{-d^2+e^2}}+\frac{2\left(2\,cd^4\left(\sqrt{x+1}-\sqrt{x-1}\right)^6e+4\,cd^5\left(\sqrt{x+1}-\sqrt{x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -(2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(1/2*((sqrt(x + 1) - s
qrt(x - 1))^2*e + 2*d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2
 + e^2)) + 2*(2*c*d^4*(sqrt(x + 1) - sqrt(x - 1))^6*e + 4*c*d^5*(sqrt(x + 1
) - sqrt(x - 1))^4 - 2*a*d^2*(sqrt(x + 1) - sqrt(x - 1))^6*e^3 - 5*c*d^2*(s
qrt(x + 1) - sqrt(x - 1))^6*e^3 + 4*b*d^4*(sqrt(x + 1) - sqrt(x - 1))^4*e +
 3*b*d*(sqrt(x + 1) - sqrt(x - 1))^6*e^4 - 12*a*d^3*(sqrt(x + 1) - sqrt(x -
```

```
 1))^4*e^2 - 14*c*d^3*(sqrt(x + 1) - sqrt(x - 1))^4*e^2 - a*(sqrt(x + 1) -
sqrt(x - 1))^6*e^5 + 10*b*d^2*(sqrt(x + 1) - sqrt(x - 1))^4*e^3 + 8*c*d^4*(
sqrt(x + 1) - sqrt(x - 1))^2*e - 6*a*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 -
8*c*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 + 16*b*d^3*(sqrt(x + 1) - sqrt(x -
1))^2*e^2 + 4*b*(sqrt(x + 1) - sqrt(x - 1))^4*e^5 - 40*a*d^2*(sqrt(x + 1) -
 sqrt(x - 1))^2*e^3 - 44*c*d^2*(sqrt(x + 1) - sqrt(x - 1))^2*e^3 + 20*b*d*(
sqrt(x + 1) - sqrt(x - 1))^2*e^4 + 8*c*d^3*e^2 + 4*a*(sqrt(x + 1) - sqrt(x
- 1))^2*e^5 + 8*b*d^2*e^3 - 24*a*d*e^4 - 32*c*d*e^4 + 16*b*e^5)/((d^4*e^2 -
 2*d^2*e^4 + e^6)*((sqrt(x + 1) - sqrt(x - 1))^4*e + 4*d*(sqrt(x + 1) - sqr
t(x - 1))^2 + 4*e)^2)
```

3.41 $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=1348

result too large to display

```
[Out] ((d*e - c*f)*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*
(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*
e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f
^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f +
 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3)))))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(512*d^5*f^5) + ((8*a^2*d^2*f^2*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*
(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d
*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 +
28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*
(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f
 + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(256*d^5*f^
4) - ((2*a*C*d*f - b*(4*B*d*f - 3*C*(d*e + c*f)))*(a + b*x)^2*(c + d*x)^(3/
2)*(e + f*x)^(3/2))/(20*b*d^2*f^2) + (C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*
x)^(3/2))/(6*b*d*f) - ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 -
8*a^2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 +
 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*
(15*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d
*f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) + (4*a*d*f - 7*b*(d*e + c*
f))*(2*a*C*d*f - b*(4*B*d*f - 3*C*(d*e + c*f))))*x))/(960*b*d^4*f^4) - ((d*
e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2
*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2
 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5
*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*Ar
cTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(512*d^(11/2)*f^(11
/2))
```

Rubi [A] time = 2.36639, antiderivative size = 1345, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 153, 147, 50, 63, 217, 206}

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^3}{6bdf}+\frac{(4bBdf-2aCdf-3bC(de+cf))(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^2}{20bd^2f^2}-\frac{(c+dx)^{3/2}(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*
(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*
e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f
^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f +
 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3)))))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(512*d^5*f^5) + ((8*a^2*d^2*f^2*(C*(5*d^2*e^2
```

```
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*
(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d
*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 +
28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*
(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f
 + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(256*d^5*f^
4) + ((4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/
2)*(e + f*x)^(3/2))/(20*b*d^2*f^2) + (C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*
x)^(3/2))/(6*b*d*f) - ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 -
8*a^2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 +
 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*
(15*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d
*f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c*
f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(960*b*d^4*f^4) - ((d*
e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2
*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2
 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5
*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*Ar
cTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(512*d^(11/2)*f^(11
/2))
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx = \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} + \frac{\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3}{2}\right.}{}$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2} +$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2} +$$

$$= \frac{\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(d\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(d\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(d\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(d\right.\right.}{}$$

Mathematica [B] time = 7.09814, size = 3599, normalized size = 2.67

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*b^2*C*(d*e - c*f)^4*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(11/2)*((63/(128*
(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f
))))^5) + 21/(32*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^4) + 63/(80*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 9/(10*(1 + (d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c +
d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/4 +
(63*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt
[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e
 - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]
*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2048*d^2*f^2*
(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))))^5)))/(3*d^5*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f)))^(9/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*b*(d*e - c*f)^3*(-4*b*C*e
 + b*B*f + 2*a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((3*(35/(64*(1
 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))
))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d*e - c*f)^2*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*
f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(512*d^2*f^2*(c + d*x)^2*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^4)
))/(3*d^4*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(d
*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)^2*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6
*a*b*C*e*f + A*b^2*f^2 + 2*a*b*B*f^2 + a^2*C*f^2)*(c + d*x)^(3/2)*Sqrt[e +
f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e
 - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/
((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt
[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*
Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(256*d^2*f^2*(c + d*x)
^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
c*f))))^3)))/(3*d^3*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/
2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*f)*(d*e - c*f)*(4*b*C*
e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*(c + d*x)^(3/2)*Sqrt[e +
 f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
 - c*f))))^(5/2)*((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c
```

```
*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*A
rcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)))])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*
x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*d^2*
f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x)
)/(d*e - c*f)]) + (2*(-(b*e) + a*f)^2*(C*e^2 - B*e*f + A*f^2)*(c + d*x)^(3/
2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)))])))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^4*Sqrt[d/
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)
])
```

Maple [B] time = 0.046, size = 6728, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.9019, size = 6765, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/30720*(15*(21*C*b^2*d^6*e^6 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)
*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b
+ A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*
(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*
b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B
*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^
5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b
+ A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 12
8*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)
*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
 d^2*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x
 + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(1280*C*b^2*d^6*f^6*x^5
+ 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)*d^6)*e^4*f^2
 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*a^2 + 2*B*a*b
+ A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*b + B*b^2)*c^2*d^4
+ 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*b)*d^6)*e^2*f^4
- 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^2)*c^3*d^3 + 56*(
C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)*c*d^5)*e*f^5 + 15*
(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)*c^4*d^2 + 40*(C*a^
2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*d^4)*f^6 + 128*(C*b
^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*f^6)*x^4 - 16*(9*C*
b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d^6)*e*f^5 + 3*(3*C*
b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)
*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 28*(2*C*a*b + B*b^2)
*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 +
2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*C*a*b + B*b^2)*c^2*
d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 + 2*A*a*b)*d^6)*f^6)*
x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*C*a*b + B*b^2)*d^6)
*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*d^5 - 100*(C*a^2 +
2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*(2*C*a*b + B*b^2)*c
^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2 + 2*A*a*b)*d^6)*e*f
^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b + B*b^2)*c^3*d^3 + 4
0*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*f^6)*x)*s
qrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6), 1/15360*(15*(21*C*b^2*d^6*e^6 - 14*(
C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*
b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*
d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*
(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*
a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2
*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b +
 B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*
b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2
)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d
^3)*f^6)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x +
c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(1280
*C*b^2*d^6*f^6*x^5 + 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b +
B*b^2)*d^6)*e^4*f^2 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 30
0*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*
b + B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A
*a*b)*d^6)*e^2*f^4 - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*
b^2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)
*c*d^5)*e*f^5 + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)
*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*
d^4)*f^6 + 128*(C*b^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*
f^6)*x^4 - 16*(9*C*b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d
^6)*e*f^5 + 3*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*
B*a*b + A*b^2)*d^6)*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 2
8*(2*C*a*b + B*b^2)*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c
*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*
```

```
C*a*b + B*b^2)*c^2*d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 +
2*A*a*b)*d^6)*f^6)*x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*
C*a*b + B*b^2)*d^6)*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*
d^5 - 100*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*
(2*C*a*b + B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2
+ 2*A*a*b)*d^6)*e*f^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b +
 B*b^2)*c^3*d^3 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*
b)*c*d^5)*f^6)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^2 \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] time = 3.68238, size = 3560, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/7680*(80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)/
(d^4*f^2) - (c*f^2 - d*f*e)/(d^4*f^4)) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*lo
g(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqr
t(d*f)*d^3*f^3))*A*a^2*abs(d)/d^2 + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^2 - (17*c*d^6*f^6 - d^7*f^5*e)/(d
^8*f^6)) + (59*c^2*d^6*f^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e^2)/(d^8*f^6)) - 3*
(5*c^3*d^6*f^6 + c^2*d^7*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f^3*e^3)/(d^8*f^6))*
sqrt(d*x + c) + 3*(5*c^4*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2*f^2*e^2 - 4*c*d^3*
f*e^3 + 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f -
c*d*f + d^2*e)))/(sqrt(d*f)*d*f^3))*C*a^2*abs(d)/d^2 + 80*(sqrt((d*x + c)*d
*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^2 - (17*c*d^6*
f^6 - d^7*f^5*e)/(d^8*f^6)) + (59*c^2*d^6*f^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e
^2)/(d^8*f^6)) - 3*(5*c^3*d^6*f^6 + c^2*d^7*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f
^3*e^3)/(d^8*f^6))*sqrt(d*x + c) + 3*(5*c^4*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2
*f^2*e^2 - 4*c*d^3*f*e^3 + 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sq
rt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^3))*B*a*b*abs(d)/d^2 + 8
*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x
+ c)/d^3 - (31*c*d^12*f^8 - d^13*f^7*e)/(d^15*f^8)) + (263*c^2*d^12*f^8 - 1
6*c*d^13*f^7*e - 7*d^14*f^6*e^2)/(d^15*f^8)) - 5*(121*c^3*d^12*f^8 - 9*c^2*
d^13*f^7*e - 9*c*d^14*f^6*e^2 - 7*d^15*f^5*e^3)/(d^15*f^8))*(d*x + c) + 15*
(7*c^4*d^12*f^8 + 2*c^3*d^13*f^7*e - 2*c*d^15*f^5*e^3 - 7*d^16*f^4*e^4)/(d^
15*f^8))*sqrt(d*x + c) - 15*(7*c^5*f^5 - 5*c^4*d*f^4*e - 2*c^3*d^2*f^3*e^2
- 2*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 + 7*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*
x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^4))*C*a*b*a
bs(d)/d^2 + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x +
c)*(6*(d*x + c)/d^2 - (17*c*d^6*f^6 - d^7*f^5*e)/(d^8*f^6)) + (59*c^2*d^6*f
```

```
^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e^2)/(d^8*f^6)) - 3*(5*c^3*d^6*f^6 + c^2*d^7
*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f^3*e^3)/(d^8*f^6))*sqrt(d*x + c) + 3*(5*c^4
*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 + 5*d^4*e^4)*log(a
bs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d
*f)*d*f^3))*A*b^2*abs(d)/d^2 + 4*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4
*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)/d^3 - (31*c*d^12*f^8 - d^13*f^7*e)/(d^
15*f^8)) + (263*c^2*d^12*f^8 - 16*c*d^13*f^7*e - 7*d^14*f^6*e^2)/(d^15*f^8)
) - 5*(121*c^3*d^12*f^8 - 9*c^2*d^13*f^7*e - 9*c*d^14*f^6*e^2 - 7*d^15*f^5*
e^3)/(d^15*f^8))*(d*x + c) + 15*(7*c^4*d^12*f^8 + 2*c^3*d^13*f^7*e - 2*c*d^
15*f^5*e^3 - 7*d^16*f^4*e^4)/(d^15*f^8))*sqrt(d*x + c) - 15*(7*c^5*f^5 - 5*
c^4*d*f^4*e - 2*c^3*d^2*f^3*e^2 - 2*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 + 7*d^5
*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e
)))/(sqrt(d*f)*d^2*f^4))*B*b^2*abs(d)/d^2 + (sqrt((d*x + c)*d*f - c*d*f + d
^2*e)*(2*(4*(2*(d*x + c)*(8*(d*x + c)*(10*(d*x + c)/d^4 - (49*c*d^20*f^10 -
 d^21*f^9*e)/(d^24*f^10)) + 3*(253*c^2*d^20*f^10 - 10*c*d^21*f^9*e - 3*d^22
*f^8*e^2)/(d^24*f^10)) - (1429*c^3*d^20*f^10 - 79*c^2*d^21*f^9*e - 49*c*d^2
2*f^8*e^2 - 21*d^23*f^7*e^3)/(d^24*f^10))*(d*x + c) + 5*(491*c^4*d^20*f^10
- 28*c^3*d^21*f^9*e - 30*c^2*d^22*f^8*e^2 - 28*c*d^23*f^7*e^3 - 21*d^24*f^6
*e^4)/(d^24*f^10))*(d*x + c) - 15*(21*c^5*d^20*f^10 + 7*c^4*d^21*f^9*e + 2*
c^3*d^22*f^8*e^2 - 2*c^2*d^23*f^7*e^3 - 7*c*d^24*f^6*e^4 - 21*d^25*f^5*e^5)
/(d^24*f^10))*sqrt(d*x + c) + 15*(21*c^6*f^6 - 14*c^5*d*f^5*e - 5*c^4*d^2*f
^4*e^2 - 4*c^3*d^3*f^3*e^3 - 5*c^2*d^4*f^2*e^4 - 14*c*d^5*f*e^5 + 21*d^6*e^
6)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))
/(sqrt(d*f)*d^3*f^5))*C*b^2*abs(d)/d^2 + 4*(sqrt((d*x + c)*d*f - c*d*f + d^
2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/(d^6*f^2) - (7*c*f^4 - d*f^3*e
)/(d^6*f^6)) + 3*(c^2*f^4 - d^2*f^2*e^2)/(d^6*f^6)) - 3*(c^3*f^3 - c^2*d*f^
2*e - c*d^2*f*e^2 + d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
 c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^5*f^4))*B*a^2*abs(d)/d^3 + 8*(sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/(d^6
*f^2) - (7*c*f^4 - d*f^3*e)/(d^6*f^6)) + 3*(c^2*f^4 - d^2*f^2*e^2)/(d^6*f^6
)) - 3*(c^3*f^3 - c^2*d*f^2*e - c*d^2*f*e^2 + d^3*e^3)*log(abs(-sqrt(d*f)*s
qrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^5*f^4))*A
*a*b*abs(d)/d^3)/d
```

3.42 $\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=721

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2+6bdfx(6aCdf-b(10Bdf-7C(cf+de)))-10abdf(8Bdf-5C(cf+de))+b^2\left(\right.\right.}{240bd^3f^3}$$

```
[Out] ((d*e - c*f)*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d
*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*
c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^
2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(128*d^4*f^4) + ((2*a*d*f*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e
^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f
) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*(c + d*x)^(3/2)*Sqrt[e + f*x])
/(64*d^4*f^3) + (C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) -
 ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f -
 5*C*(d*e + c*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*
(8*A*d*f - 5*B*(d*e + c*f))) + 6*b*d*f*(6*a*C*d*f - b*(10*B*d*f - 7*C*(d*e
+ c*f)))*x))/(240*b*d^3*f^3) - ((d*e - c*f)^2*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(9/2))
```

Rubi [A] time = 0.963188, antiderivative size = 719, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1615, 147, 50, 63, 217, 206}

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))+b^2\right.}{240bd^3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d
*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*
c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^
2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(128*d^4*f^4) + ((2*a*d*f*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e
^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f
) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*(c + d*x)^(3/2)*Sqrt[e + f*x])
/(64*d^4*f^3) + (C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) -
 ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f -
 5*C*(d*e + c*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*
(8*A*d*f - 5*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e
 + c*f))*x))/(240*b*d^3*f^3) - ((d*e - c*f)^2*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(9/2))
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf} + \frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{1}{2}b(\right.}{}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf} - \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f\right.}{}$$

$$= \frac{\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(\right.\right.}{}$$

$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+c\right.\right.\right.}{}$$

$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+c\right.\right.\right.}{}$$

$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+c\right.\right.\right.}{}$$

$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+c\right.\right.\right.}{}$$

Mathematica [B] time = 6.48294, size = 2722, normalized size = 3.78

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*b*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((3*(35/(64*(
1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d*e - c*f)^2
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[
c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(512*d^2*f^2*(c + d*x)^2*(1 + (
d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^4
)))/(3*d^4*f^3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(
d*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)^2*(-3*b*C*e + b*B*f + a*C*f)*(c
 + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(c + d*x))/((d*e
 - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(6*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (
1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c +
```

```
d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(
Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))
/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^3*(d/((d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)
*(3*b*C*e^2 - 2*b*B*e*f - 2*a*C*e*f + A*b*f^2 + a*B*f^2)*(c + d*x)^(3/2)*Sq
rt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c +
d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(
3*d^2*f^3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*f)*(C*e^2 - B*e*f + A*f^2)*(c + d*x)
^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcS
inh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f)))])))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^3*Sqr
t[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e -
c*f)])
```

Maple [B] time = 0.021, size = 3571, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] -1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-105*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^5-120*B*ln(1/2
*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*b*c^3*d^2*e*f^4+30*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*e^2*f^3+30*C*ln(1/2*(2*d*f*x+2*(
d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*
e^3*f^2+75*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^4*f-120*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+
d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e^3*f^2+75*C*ln(
1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*b*c^4*d*e*f^4-768*C*x^4*b*d^4*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*
e)^(1/2)-960*B*x^3*b*d^4*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-96
0*C*x^3*a*d^4*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-1280*A*x^2*b*
d^4*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-1280*B*x^2*a*d^4*f^4*(d
*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-60*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c
*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^2*f^3+4
```

```
80*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c^2*d^2*f^4+480*B*(d*f)^
(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*d^4*e^2*f^2-300*B*(d*f)^(1/2)*(d*f*
x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^3*d*f^4-300*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*b*d^4*e^3*f-960*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)
*a*c*d^3*f^4-960*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*d^4*e*f^3+
480*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^2*d^2*f^4+480*A*(d*f)
^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*d^4*e^2*f^2-960*A*ln(1/2*(2*d*f*x+
2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4
*e*f^4+240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e*f^4+240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^2*f^3-1920*A
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*d^4*f^4-300*C*(d*f)^(1/2)*
(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c^3*d*f^4-300*C*(d*f)^(1/2)*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*a*d^4*e^3*f-120*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x
+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^3*f^2-120*C*ln(1/2*
(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*a*c^3*d^2*e*f^4-60*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e^2*f^3-80*B*(d*f)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)*x*b*c*d^3*e*f^3-80*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*x*a*c*d^3*e*f^3+44*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*x*b*c^2*d^2*e*f^3+44*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*
c*d^3*e^2*f^2-32*C*x^2*b*c*d^3*e*f^3*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)+150*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*b*c^4*d*f^5-160*C*x^2*a*c*d^3*f^4*(d*f)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)-160*C*x^2*a*d^4*e*f^3*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*
e*x+c*e)^(1/2)+112*C*x^2*b*c^2*d^2*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e
)^(1/2)+112*C*x^2*b*d^4*e^2*f^2*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)
-105*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*b*c^5*f^5+480*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*f^5+480*A*ln(1/2*(2*d*f*
x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5
*e^2*f^3-240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*f^5+210*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*
x+c*e)^(1/2)*b*c^4*f^4+210*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*
d^4*e^4+240*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e*f^4+240*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f
*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e^2*f^3-80*C*
(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c*d^3*e^3*f+140*C*(d*f)^(1/2)
*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c^2*d^2*e*f^3+140*C*(d*f)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)*a*c*d^3*e^2*f^2+140*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*
e*x+c*e)^(1/2)*b*c^2*d^2*e*f^3+140*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*b*c*d^3*e^2*f^2+200*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a
*d^4*e^2*f^2-140*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*c^3*d*f^
4-140*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*d^4*e^3*f-320*A*(d*
f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c*d^3*e*f^3-320*B*(d*f)^(1/2)*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c*d^3*e*f^3+200*C*(d*f)^(1/2)*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*x*a*c^2*d^2*f^4+200*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)*x*b*c^2*d^2*f^4+200*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)
*x*b*d^4*e^2*f^2-320*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*c*d^
3*f^4-320*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*d^4*e*f^3-320*B
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*c*d^3*f^4-320*B*(d*f)^(1/2
)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*d^4*e*f^3-96*C*x^3*b*c*d^3*f^4*(d*f)^
(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-96*C*x^3*b*d^4*e*f^3*(d*f)^(1/2)*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)-160*B*x^2*b*c*d^3*f^4*(d*f)^(1/2)*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)-160*B*x^2*b*d^4*e*f^3*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)-80*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^3*d*e*f^3-68
*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^2*d^2*e^2*f^2+150*B*ln(1
/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*b*d^5*e^4*f+150*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d
```

```
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^4*d*f^5+150*C*ln(1/2*(2*d*f*x+2*(d*f*x^
2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^4*f-240*
B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a*c^3*d^2*f^5-240*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^3*f^2-240*A*ln(1/2*(2*d*f*x
+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*
e^3*f^2)/(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)/d^4/f^4/(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 3.78587, size = 3633, normalized size = 5.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="f
ricas")

[Out] [-1/7680*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f - 2*(
C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*c^3
*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3 -
(5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^
3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a + A
*b)*c^3*d^2)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f
^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2
*e*f + c*d*f^2)*x) - 4*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b
*c*d^4 + 15*(C*a + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c
*d^4 - 120*(B*a + A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C
*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*
c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*
e*f^4 + (C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 -
2*(C*b*c*d^4 + 5*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c
*d^4 - 80*(B*a + A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4
 + 50*(C*a + B*b)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 8
0*(B*a + A*b)*d^5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c
^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^
5), -1/3840*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f -
2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*
c^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3
 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2
*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a
+ A*b)*c^3*d^2)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x
)) - 2*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b*c*d^4 + 15*(C*a
 + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c*d^4 - 120*(B*a
```

```
+ A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C*a + B*b)*c^2*d^
3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*c*d^4 - 10*(C*a
+ B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*e*f^4 + (C*b*c*d
^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 - 2*(C*b*c*d^4 + 5
*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c*d^4 - 80*(B*a +
 A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4 + 50*(C*a + B*b
)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 80*(B*a + A*b)*d^
5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c^2*d^3 + 16*(B*a
 + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx)\sqrt{c + dx}\sqrt{e + fx}\left(A + Bx + Cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] time = 3.57262, size = 2006, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/1920*(20*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)/
(d^4*f^2) - (c*f^2 - d*f*e)/(d^4*f^4)) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*lo
g(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqr
t(d*f)*d^3*f^3))*A*a*abs(d)/d^2 + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(
2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^2 - (17*c*d^6*f^6 - d^7*f^5*e)/(d^8
*f^6)) + (59*c^2*d^6*f^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e^2)/(d^8*f^6)) - 3*(5
*c^3*d^6*f^6 + c^2*d^7*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f^3*e^3)/(d^8*f^6))*sq
rt(d*x + c) + 3*(5*c^4*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2*f^2*e^2 - 4*c*d^3*f*
e^3 + 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*
d*f + d^2*e)))/(sqrt(d*f)*d*f^3))*C*a*abs(d)/d^2 + 10*(sqrt((d*x + c)*d*f -
 c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^2 - (17*c*d^6*f^6
- d^7*f^5*e)/(d^8*f^6)) + (59*c^2*d^6*f^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e^2)/
(d^8*f^6)) - 3*(5*c^3*d^6*f^6 + c^2*d^7*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f^3*e
^3)/(d^8*f^6))*sqrt(d*x + c) + 3*(5*c^4*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2*f^2
*e^2 - 4*c*d^3*f*e^3 + 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((
d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^3))*B*b*abs(d)/d^2 + (sqrt((
d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)/d^3
 - (31*c*d^12*f^8 - d^13*f^7*e)/(d^15*f^8)) + (263*c^2*d^12*f^8 - 16*c*d^13
*f^7*e - 7*d^14*f^6*e^2)/(d^15*f^8)) - 5*(121*c^3*d^12*f^8 - 9*c^2*d^13*f^7
*e - 9*c*d^14*f^6*e^2 - 7*d^15*f^5*e^3)/(d^15*f^8))*(d*x + c) + 15*(7*c^4*d
^12*f^8 + 2*c^3*d^13*f^7*e - 2*c*d^15*f^5*e^3 - 7*d^16*f^4*e^4)/(d^15*f^8))
*sqrt(d*x + c) - 15*(7*c^5*f^5 - 5*c^4*d*f^4*e - 2*c^3*d^2*f^3*e^2 - 2*c^2*
d^3*f^2*e^3 - 5*c*d^4*f*e^4 + 7*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) +
 sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^4))*C*b*abs(d)/d^2
+ (sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x +
```

```
 c)/(d^6*f^2) - (7*c*f^4 - d*f^3*e)/(d^6*f^6)) + 3*(c^2*f^4 - d^2*f^2*e^2)/
(d^6*f^6)) - 3*(c^3*f^3 - c^2*d*f^2*e - c*d^2*f*e^2 + d^3*e^3)*log(abs(-sqr
t(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^5
*f^4))*B*a*abs(d)/d^3 + (sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*
(2*(d*x + c)*(4*(d*x + c)/(d^6*f^2) - (7*c*f^4 - d*f^3*e)/(d^6*f^6)) + 3*(c
^2*f^4 - d^2*f^2*e^2)/(d^6*f^6)) - 3*(c^3*f^3 - c^2*d*f^2*e - c*d^2*f*e^2 +
 d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d
^2*e)))/(sqrt(d*f)*d^5*f^4))*A*b*abs(d)/d^3)/d
```

3.43 $\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=330

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{32d^3f^2}+\frac{\sqrt{c+dx}\sqrt{e+fx}(de-cf)\left(8df(2Adf-\right.}{6}$$

```
[Out] ((d*e - c*f)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^3) + ((C*(5*d^2*e^2 + 6*
c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(32*d^3*f^2) - ((5*C*d*e + 11*c*C*f - 8*B*d*f)*(c + d*x)^(3/2)*
(e + f*x)^(3/2))/(24*d^2*f^2) + (C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*
f) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])]
)/(64*d^(7/2)*f^(7/2))
```

Rubi [A] time = 0.297592, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{32d^3f^2}+\frac{\sqrt{c+dx}\sqrt{e+fx}(de-cf)\left(8df(2Adf-\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^3) + ((C*(5*d^2*e^2 + 6*
c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(32*d^3*f^2) - ((5*C*d*e + 11*c*C*f - 8*B*d*f)*(c + d*x)^(3/2)*
(e + f*x)^(3/2))/(24*d^2*f^2) + (C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*
f) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])]
)/(64*d^(7/2)*f^(7/2))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx = \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{\int \sqrt{c+dx}\sqrt{e+fx}\left(\frac{1}{2}\left(-5cCde-3c^2Cf+8Ad^2f\right)-\right.}{4d^2f}$$

$$= -\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{(C}{}$$

$$= \frac{\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} -$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e-}}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e-}}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e-}}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e-}}{64d^3f^3}$$

Mathematica [A] time = 1.84877, size = 306, normalized size = 0.93

$$d\sqrt{f}\sqrt{c+dx}(e+fx)\left(8df\left(6Adf(cf+d(e+2fx))+B\left(-3c^2f^2+2cdf(e+fx)+d^2\left(-3e^2+2efx+8f^2x^2\right)\right)\right)+C\left(-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x
) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e
*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*
f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) - 3*(d*e -
c*f)^(5/2)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e
 + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[d*e - c*f]])/(192*d^4*f^(7/2)*Sqrt[e + f*x])
```

Maple [B] time = 0.016, size = 1431, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] -1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(48*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)*d^3*e^2*f+48*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c^2*d*
f^3-96*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*d^3*e*f^2-96*A*(d*f)^(
1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c*d^2*f^3-6*C*ln(1/2*(2*d*f*x+2*(d*f*x
^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*f^2
-12*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*c*d^3*e^3*f-96*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e*f^3-192*A*(d*f)^(1/2)*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*d^3*f^3+24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e*f^3+24*B*ln(1
/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*c*d^3*e^2*f^2-12*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^3*d*e*f^3-96*C*x^3*d^3*f^3*(d*f)^(1/2)*(
d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-128*B*x^2*d^3*f^3*(d*f)^(1/2)*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)-30*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*d^3*e^3-
24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*c^3*d*f^4-24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-30*C*(d*f)^(1/2)*(d*f*x^2+c
*f*x+d*e*x+c*e)^(1/2)*c^3*f^3+48*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*f^4+48*A*ln(1/2*(2*d*f*
x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e
^2*f^2-32*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*c*d^2*f^3-32*B*(d
*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*d^3*e*f^2+20*C*(d*f)^(1/2)*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*c^2*d*f^3+20*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*x*d^3*e^2*f-32*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*
c*d^2*e*f^2+14*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c^2*d*e*f^2+14
*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c*d^2*e^2*f-16*C*x^2*c*d^2*f
^3*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)-16*C*x^2*d^3*e*f^2*(d*f)^(1/
2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)+15*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*
e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+15*C*ln(1/2*(2*d*f
```

```
*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*
e^4-8*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*c*d^2*e*f^2)/(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)/d^3/f^3/(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54313, size = 1843, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*c^2*d^2 - 4*B*c
*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*e*f^3 + (5*
C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^
2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*s
qrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^
3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4
)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (
C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*
f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x +
 e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*
c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d
^3)*e*f^3 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2
*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2
+ c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^3*f
- (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4)*e*
f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (C*c*
d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*f^3
+ (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))
/(d^4*f^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx}\sqrt{e + fx}\left(A + Bx + Cx^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [B] time = 1.58253, size = 856, normalized size = 2.59

$$\frac{20\left(\sqrt{(dx+c)df-cdf+d^2e}\sqrt{dx+c}\left(\frac{2(dx+c)}{d^4f^2}-\frac{cf^2-dfe}{d^4f^4}\right)+\frac{(c^2f^2-2cdfe+d^2e^2)\log\left(\left|-\sqrt{df}\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}d^3f^3}\right)A|d|}{d^2}+\frac{10\left(\sqrt{(dx+c)df-cdf+d^2e}\left(2(dx+c)\left(4(dx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(20*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)/
(d^4*f^2) - (c*f^2 - d*f*e)/(d^4*f^4)) + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*lo
g(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqr
t(d*f)*d^3*f^3))*A*abs(d)/d^2 + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*
(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^2 - (17*c*d^6*f^6 - d^7*f^5*e)/(d^8*f
^6)) + (59*c^2*d^6*f^6 - 6*c*d^7*f^5*e - 5*d^8*f^4*e^2)/(d^8*f^6)) - 3*(5*c
^3*d^6*f^6 + c^2*d^7*f^5*e - c*d^8*f^4*e^2 - 5*d^9*f^3*e^3)/(d^8*f^6))*sqrt
(d*x + c) + 3*(5*c^4*f^4 - 4*c^3*d*f^3*e - 2*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^
3 + 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*
f + d^2*e)))/(sqrt(d*f)*d*f^3))*C*abs(d)/d^2 + (sqrt((d*x + c)*d*f - c*d*f
+ d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/(d^6*f^2) - (7*c*f^4 - d*f
^3*e)/(d^6*f^6)) + 3*(c^2*f^4 - d^2*f^2*e^2)/(d^6*f^6)) - 3*(c^3*f^3 - c^2*
d*f^2*e - c*d^2*f*e^2 + d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d
*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^5*f^4))*B*abs(d)/d^3)/d
```

3.44 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}\,dx$

Optimal. Leaf size=450

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(-8a^2bd^2f^2(2Bdf+cCf+Cde)+16a^3Cd^3f^3-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)+\cdots\right.}{8b^4d^{5/2}f^{5/2}}$$

```
[Out] ((4*b*d*f*(2*A*b*d*f - a*C*(d*e + c*f)) + (b*d*e - b*c*f + 4*a*d*f)*(2*a*C*
d*f + b*(C*d*e + c*C*f - 2*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(8*b^3*d^2
*f^2) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^
(3/2))/(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) - ((16
*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C
*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d
*e + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f
]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*b^4*d^(5/2)*f^(5/2)) - (2*(A*
b^2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*
f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b^4
```

Rubi [A] time = 1.36945, antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1615, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(-8a^2bd^2f^2(2Bdf+cCf+Cde)+16a^3Cd^3f^3-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)+\cdots\right.}{8b^4d^{5/2}f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((8*A*b*d*f - 4*a*C*(d*e + c*f) + ((b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b
*(C*d*e + c*C*f - 2*B*d*f)))/(b*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(8*b^2*d
*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3
/2))/(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) - ((16*a
^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*e
 + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*b^4*d^(5/2)*f^(5/2)) - (2*(A*b^
2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b^4
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}\,dx = \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(\frac{3}{2}b(2Abdf-aC(de+cf))-\frac{3}{2}b(2aCdf+b(Cde+cCf-2B}{a+bx}}{3b^2df}$$

$$= -\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)^{3/2}(e+fx)^{3/}}{3bdf}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2}{8b^3d^2f^2}$$

Mathematica [B] time = 6.19865, size = 1944, normalized size = 4.32

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] (2*(A*b^2 - a*b*B + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(1/(2*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
)) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSi
nh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c +
 d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2))))/
(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))
/(d*e - c*f)]) + (2*C*(d*e - c*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(5/2)*
((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*
e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
(c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*S
qrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f)))])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*b*d^2*f*(d/((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) +
 (2*(-(b*C*e) + b*B*f - a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c +
 d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(3/
```

```
(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*(
(2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
 - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[
d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c
 + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(16*
d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f)))))))/(3*b^2*d*f*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) - ((A*b^2 - a*b*B + a^2*C)
*(-(b*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[(
d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e
 - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(b*d^(3/2)*Sqrt[
e + f*x]) - (2*(-(b*e) + a*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f])))/b^3
```

Maple [B] time = 0.044, size = 4227, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x)
```

```
[Out] -1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-48*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b^3*c*d^2*f^3-48*A*ln((-2*a
*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b
^3*d^3*e*f^2+48*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*(d*f)^(1/2)*b^4*c*d^2*e*f^2+24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*c*d^2*f^3+24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f
*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*d^3*e*f^2-12*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*
f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c*d^2*e*f^2+48*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e
*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*c*d^2*f^3+48*B*
ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(
1/2)*a^2*b^2*d^3*e*f^2-24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(
d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*d^2*f^2-24*C*ln(1/2*(2*d*f
*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*c*d^2*f^3-24*C*ln(1/2*(2*
d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*d^3*e*f^2-6*C*ln(1/2*(
2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*c^2*d*f^3-6*C*ln(1/2*(
2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*d^3*e^2*f+3*C*ln(1/2*(
2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c^2*d*e*f^2+3*C*ln(1/2*(
2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c*d^2*e^2*f-16*C*x^2*b^4
```

```
*d^2*f^2*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)+12*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*b^3*c*d*f^2+12*C*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/
2)*a*b^3*d^2*e*f-4*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1
/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*c*d*e*f+24*C*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*b^3
*d^2*f^2-4*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*c*d*f^2+48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e
*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*c*d^2*e*f^2-3*C
*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c^3*f^3-48*A*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*
x+c*e)^(1/2)*b^4*d^2*f^2+6*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*c^2*f^2+6*C*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*
b^4*d^2*e^2+48*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b
^3*d^3*f^3-24*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*
c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d
^3*e*f^2-48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*(d*f)^(1/2)*a^3*b*c*d^2*f^3-48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)
*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^3*b*d^3*e*f^2+48*B*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*a*b^3*d^2*f^2-12*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f
)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*c*d*f^2-12*B*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4
*d^2*e*f-48*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1/2)*(d*
f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a^2*b^2*d^2*f^2-3*C*ln(1/2*(2*d*f*x+2*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^3*e^3+48*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*
x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^4*d^3*f^3-48*B*ln((-2*
a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*
f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*
b^3*c*d^2*e*f^2+12*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*a*b^3*c*d^2*e*f^2-4*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^
(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*d^2*e*f+48*A*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*d^3*f
^3-48*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*d^3*
f^3+6*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c^2*d*f^
3+6*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^3*e^2*f-
48*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d
*f)^(1/2)*a^3*b*d^3*f^3+48*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*a^3*b*d^3*f^3)/(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)/b^5/d^2/f^2/(d*f)^(
1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

Giac [B] time = 3.32303, size = 1461, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] 1/24*sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x
 + c)*C*abs(d)/(b*d^4) - (7*C*b^9*c*d^8*f^4*abs(d) + 6*C*a*b^8*d^9*f^4*abs(
d) - 6*B*b^9*d^9*f^4*abs(d) - C*b^9*d^9*f^3*abs(d)*e)/(b^10*d^12*f^4)) + 3*
(C*b^9*c^2*d^8*f^4*abs(d) + 2*C*a*b^8*c*d^9*f^4*abs(d) - 2*B*b^9*c*d^9*f^4*
abs(d) + 8*C*a^2*b^7*d^10*f^4*abs(d) - 8*B*a*b^8*d^10*f^4*abs(d) + 8*A*b^9*
d^10*f^4*abs(d) - 2*C*a*b^8*d^10*f^3*abs(d)*e + 2*B*b^9*d^10*f^3*abs(d)*e -
 C*b^9*d^10*f^2*abs(d)*e^2)/(b^10*d^12*f^4)) + 2*(sqrt(d*f)*C*a^3*b*c*f*abs
(d) - sqrt(d*f)*B*a^2*b^2*c*f*abs(d) + sqrt(d*f)*A*a*b^3*c*f*abs(d) - sqrt(
```

```
d*f)*C*a^4*d*f*abs(d) + sqrt(d*f)*B*a^3*b*d*f*abs(d) - sqrt(d*f)*A*a^2*b^2*
d*f*abs(d) - sqrt(d*f)*C*a^2*b^2*c*abs(d)*e + sqrt(d*f)*B*a*b^3*c*abs(d)*e
- sqrt(d*f)*A*b^4*c*abs(d)*e + sqrt(d*f)*C*a^3*b*d*abs(d)*e - sqrt(d*f)*B*a
^2*b^2*d*abs(d)*e + sqrt(d*f)*A*a*b^3*d*abs(d)*e)*arctan(-1/2*(b*c*d*f - 2*
a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
 d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*
d))/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*b^4*d) - 1
/16*(sqrt(d*f)*C*b^3*c^3*f^3*abs(d) + 2*sqrt(d*f)*C*a*b^2*c^2*d*f^3*abs(d)
- 2*sqrt(d*f)*B*b^3*c^2*d*f^3*abs(d) + 8*sqrt(d*f)*C*a^2*b*c*d^2*f^3*abs(d)
 - 8*sqrt(d*f)*B*a*b^2*c*d^2*f^3*abs(d) + 8*sqrt(d*f)*A*b^3*c*d^2*f^3*abs(d
) - 16*sqrt(d*f)*C*a^3*d^3*f^3*abs(d) + 16*sqrt(d*f)*B*a^2*b*d^3*f^3*abs(d)
 - 16*sqrt(d*f)*A*a*b^2*d^3*f^3*abs(d) - sqrt(d*f)*C*b^3*c^2*d*f^2*abs(d)*e
 - 4*sqrt(d*f)*C*a*b^2*c*d^2*f^2*abs(d)*e + 4*sqrt(d*f)*B*b^3*c*d^2*f^2*abs
(d)*e + 8*sqrt(d*f)*C*a^2*b*d^3*f^2*abs(d)*e - 8*sqrt(d*f)*B*a*b^2*d^3*f^2*
abs(d)*e + 8*sqrt(d*f)*A*b^3*d^3*f^2*abs(d)*e - sqrt(d*f)*C*b^3*c*d^2*f*abs
(d)*e^2 + 2*sqrt(d*f)*C*a*b^2*d^3*f*abs(d)*e^2 - 2*sqrt(d*f)*B*b^3*d^3*f*ab
s(d)*e^2 + sqrt(d*f)*C*b^3*d^3*abs(d)*e^3)*log((sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2)/(b^4*d^4*f^3)
```

3.45 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=521

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)+b^2\left(-\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)}{4b^4d^{3/2}f^{3/2}}+\sqrt{c}$$

```
[Out] ((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A*
f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*d*f*(b*e - a*f))
 + ((3*a^2*C*d*f + b^2*(c*C*e + 2*A*d*f) - a*b*(C*d*e + c*C*f + 2*B*d*f))*S
qrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 -
 a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)
*(a + b*x)) + ((24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^
2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*b^4*d^(3/2)*f^(3/2)) + ((6*a^3*C*d
*f - b^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2
*A*d*f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[
c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[b*e -
 a*f])
```

Rubi [A] time = 1.69576, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1613, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)+b^2\left(-\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)}{4b^4d^{3/2}f^{3/2}}+\sqrt{c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```


```
[Out] ((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A*
f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*d*f*(b*e - a*f))
 + ((3*a^2*C*d*f + b^2*(c*C*e + 2*A*d*f) - a*b*(C*d*e + c*C*f + 2*B*d*f))*S
qrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 -
 a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)
*(a + b*x)) + ((24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^
2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*b^4*d^(3/2)*f^(3/2)) + ((6*a^3*C*d
*f - b^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2
*A*d*f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[
c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[b*e -
 a*f])
```


Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
```

```
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)+b^2(2Bce\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf)\right)\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)} \cdots$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf))\right)\cdots}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf))\right)\cdots}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf))\right)\cdots}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf))\right)\cdots}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf))\right)\cdots}{4b^3df(be-af)}$$

Mathematica [B] time = 6.27427, size = 2665, normalized size = 5.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x))) + (2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))
^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x
]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))))^(3/2))))/(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sq
rt[(d*(e + f*x))/(d*e - c*f)]) + (2*C*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3
/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])
/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
(d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])
))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))))))/(3*b^2*d*Sqrt[d/((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) - ((b*B - 2*a*C)*(-(b
*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f
```

```
)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*
f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(b*d^(3/2)*Sqrt[e + f
*x]) - (2*(-(b*e) + a*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c
) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f])))/b^3 - ((
A*b^2 - a*b*B + a^2*C)*((-4*f*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c +
d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(3/(
4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((
2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
- (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/
(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d
*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(16*d
^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)))))))/(3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + ((2*a*b*d*f + (b*(-2*a*d*f - b*(d
*e + c*f)))/2)*((2*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(1/(2*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (Sq
rt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt
[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2))))/(b*Sqrt[
d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]) - ((-(b*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/(d*e - c
*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]
*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(S
qrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(b*d^(3/2
)*Sqrt[e + f*x]) - (2*(-(b*e) + a*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])
/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f]
)))/b))/b))/(b^2*(b*c - a*d)*(b*e - a*f))
```

Maple [B] time = 0.042, size = 5051, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 13.1467, size = 2140, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"giac")
```

```
[Out] 1/4*sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d)
/(b^2*d^3) - (C*b^7*c*d^3*f^2*abs(d) + 8*C*a*b^6*d^4*f^2*abs(d) - 4*B*b^7*d
^4*f^2*abs(d) - C*b^7*d^4*f*abs(d)*e)/(b^9*d^6*f^2)) - (5*sqrt(d*f)*C*a^2*b
*c*f*abs(d) - 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) + sqrt(d*f)*A*b^3*c*f*abs(d) -
 6*sqrt(d*f)*C*a^3*d*f*abs(d) + 4*sqrt(d*f)*B*a^2*b*d*f*abs(d) - 2*sqrt(d*f
)*A*a*b^2*d*f*abs(d) - 4*sqrt(d*f)*C*a*b^2*c*abs(d)*e + 2*sqrt(d*f)*B*b^3*c
*abs(d)*e + 5*sqrt(d*f)*C*a^2*b*d*abs(d)*e - 3*sqrt(d*f)*B*a*b^2*d*abs(d)*e
 + sqrt(d*f)*A*b^3*d*abs(d)*e)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e -
 (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt
(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/(sqrt(a*b*c*d*f
^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*b^4*d) - 2*(sqrt(d*f)*C*a^2*b
*c^2*d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f)*A*b^3*c^
2*d*f^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*f*abs(d)*e + 2*sqrt(d*f)*B*a*b^2
*c*d^2*f*abs(d)*e - 2*sqrt(d*f)*A*b^3*c*d^2*f*abs(d)*e - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*f*abs(d
) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^2*B*a*b^2*c*f*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d*f*abs(d) - 2*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^
2*b*d*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*A*a*b^2*d*f*abs(d) + sqrt(d*f)*C*a^2*b*d^3*abs(d)*e^2 -
```

```
 sqrt(d*f)*B*a*b^2*d^3*abs(d)*e^2 + sqrt(d*f)*A*b^3*d^3*abs(d)*e^2 - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*d*abs(d)*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^2*B*a*b^2*d*abs(d)*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d*abs(d)*e)/((b*c^2*d^2*f^2 -
2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d
^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*b)*b^4) + 1/8*(sqrt(d*f)*C*b^2*c^2*f^2*abs(d) +
 8*sqrt(d*f)*C*a*b*c*d*f^2*abs(d) - 4*sqrt(d*f)*B*b^2*c*d*f^2*abs(d) - 24*s
qrt(d*f)*C*a^2*d^2*f^2*abs(d) + 16*sqrt(d*f)*B*a*b*d^2*f^2*abs(d) - 8*sqrt(
d*f)*A*b^2*d^2*f^2*abs(d) - 2*sqrt(d*f)*C*b^2*c*d*f*abs(d)*e + 8*sqrt(d*f)*
C*a*b*d^2*f*abs(d)*e - 4*sqrt(d*f)*B*b^2*d^2*f*abs(d)*e + sqrt(d*f)*C*b^2*d
^2*abs(d)*e^2)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2)/(b^4*d^3*f^2)
```

3.46 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=658

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(3a^2b^2\left(4Bdf(cf+de)+C\left(5c^2f^2+22cdef+5d^2e^2\right)\right)-8a^3bdf(Bdf+5C(cf+de))+24a^4C\cdots\right.}{4b^4(bc-\cdots}$$

```
[Out] -((12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) + a*b^2*(B*f*(5
*d*e + 3*c*f) + 4*C*e*(d*e + 4*c*f)) - b^3*(A*d*e*f + c*(4*C*e^2 + 4*B*e*f
- A*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*(b*c - a*d)*(b*e - a*f)^2) +
 ((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e +
 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C*(d*e + c*f)))*Sqrt[c + d*x]*(e +
 f*x)^(3/2))/(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B
 - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(2*b*(b*c - a*d)*(b*e - a*f)*(a +
 b*x)^2) - ((6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^4*Sqrt[d]*Sqrt[f]) - ((24*a^4*C*d^2
*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*B*f))
- 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(2*B*e +
 A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f) +
 C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c +
 d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^4*(b*c - a*d)^(3/2)*(b*e - a*
f)^(3/2))
```

Rubi [A] time = 2.67951, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1613, 149, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(3a^2b^2\left(4Bdf(cf+de)+C\left(5c^2f^2+22cdef+5d^2e^2\right)\right)-8a^3bdf(Bdf+5C(cf+de))+24a^4C\cdots\right.}{4b^4(bc-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -((12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*C*e^
2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(d*e
+ 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*(b*c - a*d)*(b*e - a*f)^2) +
 ((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e +
 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C*(d*e + c*f)))*Sqrt[c + d*x]*(e +
 f*x)^(3/2))/(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B
 - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(2*b*(b*c - a*d)*(b*e - a*f)*(a +
 b*x)^2) - ((6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^4*Sqrt[d]*Sqrt[f]) - ((24*a^4*C*d^2
*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*B*f))
- 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(2*B*e +
 A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f) +
 C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c +
 d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^4*(b*c - a*d)^(3/2)*(b*e - a*
f)^(3/2))
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
```

```
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)+b^2(4Bce\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(6a^3Cdf-b^3(4Bce-Ade-Acf)+ab^2(8cCe+3Bde+3Bcf-2Adf)-a^2\cdots\right.}{4b^2(bc-ad)(be-af)^2(a+bx)\cdots}$$

$$= -\frac{\left(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)-b^3\left(4cCe^2+Adef+cf(4Bc\cdots\right.\right.}{4b^3(bc-ad)(be\cdots}$$

$$= -\frac{\left(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)-b^3\left(4cCe^2+Adef+cf(4Bc\cdots\right.\right.}{4b^3(bc-ad)(be\cdots}$$

$$= -\frac{\left(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)-b^3\left(4cCe^2+Adef+cf(4Bc\cdots\right.\right.}{4b^3(bc-ad)(be\cdots}$$

$$= -\frac{\left(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)-b^3\left(4cCe^2+Adef+cf(4Bc\cdots\right.\right.}{4b^3(bc-ad)(be\cdots}$$

$$= -\frac{\left(12a^3Cdf^2-a^2bf(17Cde+11cCf+4Bdf)-b^3\left(4cCe^2+Adef+cf(4Bc\cdots\right.\right.}{4b^3(bc-ad)(be\cdots}$$

Mathematica [B] time = 6.44909, size = 2157, normalized size = 3.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*e - a*f)
*(a + b*x)^2) - ((b*B - 2*a*C)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)) + (2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(1
/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3
/2))))/(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]) + (2*C*(b*c - a*d)*((Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[(d*
```

```
(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(
b*d*Sqrt[e + f*x]) + (Sqrt[b*e - a*f]*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x]
)/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d])))/b^3 - ((A*b
^2 - a*(b*B - a*C))*(d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)
*(a + b*x)) - ((d*e - c*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b
*c) + a*d]*Sqrt[e + f*x])])/((-(b*c) + a*d)^(3/2)*Sqrt[b*e - a*f])))/(4*b^2
*(b*e - a*f)) - ((b*B - 2*a*C)*((-4*f*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3
/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])
/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
(d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])
))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f)))))))/(3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)
/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + ((2*a*b*d*f + (b*(-2*a*d*
f - b*(d*e + c*f)))/2)*((2*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(1/(2*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
)) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSi
nh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c +
 d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2))))/
(b*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(
d*e - c*f)]) - ((-(b*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
 - c*f)]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c +
d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(
b*d^(3/2)*Sqrt[e + f*x]) - (2*(-(b*e) + a*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d]*Sqrt[b*
e - a*f])))/b))/b))/(b^2*(b*c - a*d)*(b*e - a*f))
```

Maple [B] time = 0.063, size = 12065, normalized size = 18.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 38.8443, size = 11268, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"giac")
```

```
[Out] 1/4*(15*sqrt(d*f)*C*a^2*b^2*c^2*f^2*abs(d) - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*ab
s(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs(d) - 40*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d)
 + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2*abs(d) + 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d
) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c^2*f*abs(d)*
e + 4*sqrt(d*f)*B*b^4*c^2*f*abs(d)*e + 66*sqrt(d*f)*C*a^2*b^2*c*d*f*abs(d)*
e - 18*sqrt(d*f)*B*a*b^3*c*d*f*abs(d)*e + 2*sqrt(d*f)*A*b^4*c*d*f*abs(d)*e
- 40*sqrt(d*f)*C*a^3*b*d^2*f*abs(d)*e + 12*sqrt(d*f)*B*a^2*b^2*d^2*f*abs(d)
*e + 8*sqrt(d*f)*C*b^4*c^2*abs(d)*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d*abs(d)*e^2
 + 4*sqrt(d*f)*B*b^4*c*d*abs(d)*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^2*abs(d)*e^2
 - 3*sqrt(d*f)*B*a*b^3*d^2*abs(d)*e^2 - sqrt(d*f)*A*b^4*d^2*abs(d)*e^2)*arc
tan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c
*d*f*e + a*b*d^2*f*e)*d))/((a*b^5*c*f - a^2*b^4*d*f - b^6*c*e + a*b^5*d*e)*
sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 1/2*(9*sqr
t(d*f)*C*a^2*b^3*c^5*d^3*f^5*abs(d) - 5*sqrt(d*f)*B*a*b^4*c^5*d^3*f^5*abs(d
) + sqrt(d*f)*A*b^5*c^5*d^3*f^5*abs(d) - 10*sqrt(d*f)*C*a^3*b^2*c^4*d^4*f^5
*abs(d) + 6*sqrt(d*f)*B*a^2*b^3*c^4*d^4*f^5*abs(d) - 2*sqrt(d*f)*A*a*b^4*c^
4*d^4*f^5*abs(d) - 8*sqrt(d*f)*C*a*b^4*c^5*d^3*f^4*abs(d)*e + 4*sqrt(d*f)*B
*b^5*c^5*d^3*f^4*abs(d)*e - 27*sqrt(d*f)*C*a^2*b^3*c^4*d^4*f^4*abs(d)*e + 1
5*sqrt(d*f)*B*a*b^4*c^4*d^4*f^4*abs(d)*e - 3*sqrt(d*f)*A*b^5*c^4*d^4*f^4*ab
s(d)*e + 40*sqrt(d*f)*C*a^3*b^2*c^3*d^5*f^4*abs(d)*e - 24*sqrt(d*f)*B*a^2*b
^3*c^3*d^5*f^4*abs(d)*e + 8*sqrt(d*f)*A*a*b^4*c^3*d^5*f^4*abs(d)*e - 27*sqr
```

t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*
a^2*b^3*c^4*d^2*f^4*abs(d) + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^4*d^2*f^4*abs(d) - 3*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^4*
d^2*f^4*abs(d) + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^3*f^4*abs(d) - 44*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^3*d^3*f
^4*abs(d) + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^2*A*a*b^4*c^3*d^3*f^4*abs(d) - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^4*f^4*abs(d)
+ 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*B*a^3*b^2*c^2*d^4*f^4*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c^2*d^4*f^4*abs(d) + 32*s
qrt(d*f)*C*a*b^4*c^4*d^4*f^3*abs(d)*e^2 - 16*sqrt(d*f)*B*b^5*c^4*d^4*f^3*ab
s(d)*e^2 + 18*sqrt(d*f)*C*a^2*b^3*c^3*d^5*f^3*abs(d)*e^2 - 10*sqrt(d*f)*B*a
*b^4*c^3*d^5*f^3*abs(d)*e^2 + 2*sqrt(d*f)*A*b^5*c^3*d^5*f^3*abs(d)*e^2 - 60
*sqrt(d*f)*C*a^3*b^2*c^2*d^6*f^3*abs(d)*e^2 + 36*sqrt(d*f)*B*a^2*b^3*c^2*d^
6*f^3*abs(d)*e^2 - 12*sqrt(d*f)*A*a*b^4*c^2*d^6*f^3*abs(d)*e^2 + 24*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^
4*c^4*d^2*f^3*abs(d)*e - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
+ c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^4*d^2*f^3*abs(d)*e - 44*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3
*d^3*f^3*abs(d)*e + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2*B*a*b^4*c^3*d^3*f^3*abs(d)*e + 4*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^3*d^3*f^3
*abs(d)*e - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*C*a^3*b^2*c^2*d^4*f^3*abs(d)*e + 44*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^2*d^4*f^3*
abs(d)*e - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^2*A*a*b^4*c^2*d^4*f^3*abs(d)*e + 112*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c*d^5*f^3*abs(d)*
e - 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^2*B*a^3*b^2*c*d^5*f^3*abs(d)*e + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^5*f^3*abs(d)*e + 2
7*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^4*C*a^2*b^3*c^3*d*f^3*abs(d) - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^3*d*f^3*abs(d) + 3*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3
*d*f^3*abs(d) - 102*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^4*C*a^3*b^2*c^2*d^2*f^3*abs(d) + 58*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^2*b^3*c^2*d^2*f
^3*abs(d) - 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^4*A*a*b^4*c^2*d^2*f^3*abs(d) + 152*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*c*d^3*f^3*abs(d)
- 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^4*B*a^3*b^2*c*d^3*f^3*abs(d) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*c*d^3*f^3*abs(d) - 80*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a
^5*d^4*f^3*abs(d) + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^4*B*a^4*b*d^4*f^3*abs(d) - 16*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^3*b^2*d^4*f^3*abs(
d) - 48*sqrt(d*f)*C*a*b^4*c^3*d^5*f^2*abs(d)*e^3 + 24*sqrt(d*f)*B*b^5*c^3*d
^5*f^2*abs(d)*e^3 + 18*sqrt(d*f)*C*a^2*b^3*c^2*d^6*f^2*abs(d)*e^3 - 10*sqrt
(d*f)*B*a*b^4*c^2*d^6*f^2*abs(d)*e^3 + 2*sqrt(d*f)*A*b^5*c^2*d^6*f^2*abs(d)
*e^3 + 40*sqrt(d*f)*C*a^3*b^2*c*d^7*f^2*abs(d)*e^3 - 24*sqrt(d*f)*B*a^2*b^3
*c*d^7*f^2*abs(d)*e^3 + 8*sqrt(d*f)*A*a*b^4*c*d^7*f^2*abs(d)*e^3 - 24*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*
b^4*c^3*d^3*f^2*abs(d)*e^2 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^3*d^3*f^2*abs(d)*e^2 + 142*sqrt(d*

```
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*
b^3*c^2*d^4*f^2*abs(d)*e^2 - 70*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^2*d^4*f^2*abs(d)*e^2 - 2*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*
c^2*d^4*f^2*abs(d)*e^2 - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
+ c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c*d^5*f^2*abs(d)*e^2 + 44*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3
*c*d^5*f^2*abs(d)*e^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*c*d^5*f^2*abs(d)*e^2 - 56*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*d^6*f
^2*abs(d)*e^2 + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*B*a^3*b^2*d^6*f^2*abs(d)*e^2 - 8*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*d^6*f^2*abs
(d)*e^2 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^4*C*a*b^4*c^3*d*f^2*abs(d)*e + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*b^5*c^3*d*f^2*abs(d)*e + 1
09*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^4*C*a^2*b^3*c^2*d^2*f^2*abs(d)*e - 57*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^2*d^2*f^2*abs(d)*e + 5*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*
A*b^5*c^2*d^2*f^2*abs(d)*e - 228*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c*d^3*f^2*abs(d)*e + 124*sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^2
*b^3*c*d^3*f^2*abs(d)*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
 + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c*d^3*f^2*abs(d)*e + 152*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*d^4
*f^2*abs(d)*e - 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^4*B*a^3*b^2*d^4*f^2*abs(d)*e + 24*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*d^4*f^2*abs(
d)*e - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^6*C*a^2*b^3*c^2*f^2*abs(d) + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a*b^4*c^2*f^2*abs(d) - sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*b^5*c^2
*f^2*abs(d) + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^6*C*a^3*b^2*c*d*f^2*abs(d) - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a^2*b^3*c*d*f^2*abs(d) +
 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^6*A*a*b^4*c*d*f^2*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^4*b*d^2*f^2*abs(d) + 16*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a^3*b^2*d^2
*f^2*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*A*a^2*b^3*d^2*f^2*abs(d) + 32*sqrt(d*f)*C*a*b^4*c^2*d^6*f*
abs(d)*e^4 - 16*sqrt(d*f)*B*b^5*c^2*d^6*f*abs(d)*e^4 - 27*sqrt(d*f)*C*a^2*b
^3*c*d^7*f*abs(d)*e^4 + 15*sqrt(d*f)*B*a*b^4*c*d^7*f*abs(d)*e^4 - 3*sqrt(d*
f)*A*b^5*c*d^7*f*abs(d)*e^4 - 10*sqrt(d*f)*C*a^3*b^2*d^8*f*abs(d)*e^4 + 6*s
qrt(d*f)*B*a^2*b^3*d^8*f*abs(d)*e^4 - 2*sqrt(d*f)*A*a*b^4*d^8*f*abs(d)*e^4
- 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*C*a*b^4*c^2*d^4*f*abs(d)*e^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^2*d^4*f*abs(d)*e^3 - 44*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C
*a^2*b^3*c*d^5*f*abs(d)*e^3 + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c*d^5*f*abs(d)*e^3 + 4*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c*d
^5*f*abs(d)*e^3 + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*
f - c*d*f + d^2*e))^2*C*a^3*b^2*d^6*f*abs(d)*e^3 - 44*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*d^6*f*abs(
d)*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*A*a*b^4*d^6*f*abs(d)*e^3 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a*b^4*c^2*d^2*f*abs(d)*e^2 + 8
```

```
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^
4*B*b^5*c^2*d^2*f*abs(d)*e^2 + 109*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c*d^3*f*abs(d)*e^2 - 57*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*
b^4*c*d^3*f*abs(d)*e^2 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*A*b^5*c*d^3*f*abs(d)*e^2 - 102*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*d^4*f
*abs(d)*e^2 + 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^4*B*a^2*b^3*d^4*f*abs(d)*e^2 - 14*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*d^4*f*abs(d)*e^2
 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^6*C*a*b^4*c^2*f*abs(d)*e - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^6*B*b^5*c^2*f*abs(d)*e - 38*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^2*b^3*c*d*
f*abs(d)*e + 22*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*B*a*b^4*c*d*f*abs(d)*e - 6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*b^5*c*d*f*abs(d)*e + 32*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a
^3*b^2*d^2*f*abs(d)*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^6*B*a^2*b^3*d^2*f*abs(d)*e + 8*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*a*b^4*d^2*f*abs
(d)*e - 8*sqrt(d*f)*C*a*b^4*c*d^7*abs(d)*e^5 + 4*sqrt(d*f)*B*b^5*c*d^7*abs(
d)*e^5 + 9*sqrt(d*f)*C*a^2*b^3*d^8*abs(d)*e^5 - 5*sqrt(d*f)*B*a*b^4*d^8*abs
(d)*e^5 + sqrt(d*f)*A*b^5*d^8*abs(d)*e^5 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c*d^5*abs(d)*e^4 - 1
2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*B*b^5*c*d^5*abs(d)*e^4 - 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d
*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*d^6*abs(d)*e^4 + 15*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*d^6*
abs(d)*e^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*A*b^5*d^6*abs(d)*e^4 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a*b^4*c*d^3*abs(d)*e^3 + 12*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B
*b^5*c*d^3*abs(d)*e^3 + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*d^4*abs(d)*e^3 - 15*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*d^4*abs(
d)*e^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^4*A*b^5*d^4*abs(d)*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a*b^4*c*d*abs(d)*e^2 - 4*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*b^5*c*d
*abs(d)*e^2 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*C*a^2*b^3*d^2*abs(d)*e^2 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a*b^4*d^2*abs(d)*e^2 - sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*
b^5*d^2*abs(d)*e^2)/((a*b^5*c*f - a^2*b^4*d*f - b^6*c*e + a*b^5*d*e)*(b*c^2
*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)^2) + sqrt((d*x + c)*d*f - c*d*f +
 d^2*e)*sqrt(d*x + c)*C*abs(d)/(b^3*d^2) - 1/2*(sqrt(d*f)*C*b*c*f*abs(d) -
6*sqrt(d*f)*C*a*d*f*abs(d) + 2*sqrt(d*f)*B*b*d*f*abs(d) + sqrt(d*f)*C*b*d*a
bs(d)*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^2)/(b^4*d^2*f)
```

3.47 $\int \frac{(a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1032

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^3}{5bdf}-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^2}{40bd^2f^2}-\frac{(c+dx)^{3/2}\sqrt{e+f}}{}$$

```
[Out] -((16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d
*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^
2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*
c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
 - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Sqrt[c +
d*x]*Sqrt[e + f*x])/(128*d^4*f^5) - ((4*a*C*d*f + b*(9*C*d*e + 7*c*C*f - 10
*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(40*b*d^2*f^2) + (C*(a
+ b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e
 + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*f)
 + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c
*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d*e
*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 22*
c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C*f
 - 10*A*b*d*f) - (7*b*d*e + 5*b*c*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e + 7*
c*C*f - 10*B*d*f)))*x))/(960*b*d^4*f^4) + ((d*e - c*f)*(16*a^2*d^2*f^2*(2*d
*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + 4*a
*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f
*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) - b^2*(C*(6
3*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*f^
4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3*e^3 + 15*
c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(11/2))
```

Rubi [A] time = 1.78768, antiderivative size = 1032, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 153, 147, 50, 63, 217, 206}

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^3}{5bdf}-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^2}{40bd^2f^2}-\frac{(c+dx)^{3/2}\sqrt{e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -((16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d
*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^
2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*
c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
 - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Sqrt[c +
d*x]*Sqrt[e + f*x])/(128*d^4*f^5) - ((4*a*C*d*f + b*(9*C*d*e + 7*c*C*f - 10
*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(40*b*d^2*f^2) + (C*(a
+ b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e
 + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*f)
 + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c
```

```
*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d*e
*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 22*
c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C*f
 - 10*A*b*d*f) - (7*b*d*e + 5*b*c*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e + 7*
c*C*f - 10*B*d*f)))*x))/(960*b*d^4*f^4) + ((d*e - c*f)*(16*a^2*d^2*f^2*(2*d
*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + 4*a
*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f
*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) - b^2*(C*(6
3*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*f^
4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3*e^3 + 15*
c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(11/2))
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx = \frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{\int \frac{(a+bx)^2\sqrt{c+dx}\left(-\frac{1}{2}b(6bcCe+3aCde+acCf-10Abdf)\right.}{\sqrt{e+fx}}}{5b^2df}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2} + \frac{C(a}{}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2} + \frac{C(a}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+4ab}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+4ab}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+4ab}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+4ab}{}$$

Mathematica [B] time = 6.67163, size = 3220, normalized size = 3.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((-(b*e) + a*f)^2*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)^2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e - c*f
```

```
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2*d^3*f^6*Sqrt[c + d*x]*Sqrt[e +
 f*x]) + (2*b^2*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((
3*(35/(64*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((
d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d
*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f
]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(512*d^2*f^2*(c + d*
x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))))^4)))/(3*d^4*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(
7/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*b*(d*e - c*f)^2*(-4*b*C*e + b*B*
f + 2*a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5
/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)))])))/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^4*(d/((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) +
 (2*(d*e - c*f)*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6*a*b*C*e*f + A*b^2*f^2 + 2*a*
b*B*f^2 + a^2*C*f^2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (
d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2
) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])]
)/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]
)))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*d^2*f^4*(d/((d^2*e)/(d*e - c*f) -
(c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) +
 a*f)*(4*b*C*e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*(c + d*x)^(
3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSin
h[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)))])))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/(
```

```
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^4*Sqrt[
d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*
f)])
```

Maple [B] time = 0.042, size = 3958, normalized size = 3.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)

[Out] 1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(768*C*x^4*b^2*d^4*f^4*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+1280*C*x^2*a^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+1280*A*x^2*b^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+1920*B*(d*f)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a^2*d^4*f^4+680*C*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*a*b*c^2*d^2*e*f^3+1000*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b
*c*d^3*e^2*f^2+156*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^2*d^2*e*f^
3-2240*C*x^2*a*b*d^4*e*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-128*C*x^2*b^
2*c*d^3*e*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+320*C*x^2*a*b*c*d^3*f^4*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+196*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*x*b^2*c*d^3*e^2*f^2-1280*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^3*
e*f^3-400*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c^2*d^2*f^4+2800*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*d^4*e^2*f^2-240*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*x*b^2*c*d^3*e*f^3-3200*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(
1/2)*x*a*b*d^4*e*f^3+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^5*f^5+340*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b^2*c^2*d^2*e*f^3+500*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^
3*e^2*f^2-640*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c*d^3*e*f^3+600*C*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c^3*d*f^4-220*C*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*b^2*c^3*d*e*f^3-272*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
^2*c^2*d^2*e^2*f^2-420*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^3*e^3*
f+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^3*d*f^4-1260*C*(d*f)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^4*e^3*f+1920*A*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*a*b*c*d^3*f^4-640*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d
^3*e*f^3+3840*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*d^4*f^4+320*A*(d*
f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d^3*f^4+480*B*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e*f^4-2
40*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a*b*c^3*d^2*e*f^4-360*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e^2*f^3+96*C*x^3*b^2*c*d^3*f^4*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-864*C*x^3*b^2*d^4*e*f^3*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+2560*B*x^2*a*b*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+160*B*x^2*b^2*c*d^3*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-1120*B*x^2*
b^2*d^4*e*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-112*C*x^2*b^2*c^2*d^2*f^4
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+1008*C*x^2*b^2*d^4*e^2*f^2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+1920*C*x^3*a*b*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)-1600*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^4*e*f^3-200*B*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^2*d^2*f^4+1400*B*(d*f)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*x*b^2*d^4*e^2*f^2+320*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*x*a^2*c*d^3*f^4-1600*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a^2*d^4*
e*f^3+4800*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*e^2*f^2-4200*C*(d*
f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*e^3*f-5760*A*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*a*b*d^4*e*f^3+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^4*e^2*f^3-1200*C*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^4*e^
3*f^2+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
```

```
/(d*f)^(1/2))*a^2*d^5*e^2*f^3-480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a
*b*c*d^3*e*f^3+2880*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^2*f^3+720*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^2*f^3-480*C*(d*f)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c^2*d^2*f^4+240*A*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e*f^4+480
*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*a*b*c^3*d^2*f^5-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e*f^4-180*B*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e^2*f^3+240
*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*a^2*c^2*d^3*e*f^4+960*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c*d^3*f
^4+300*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^3*d*f^4-480*A*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d^2*f^4-960*A*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*f^5+720*C*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2
*c*d^4*e^2*f^3+2100*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^4*f+525*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^4*f+2400*A*(d*f)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^4*e^2*f^2-2880*B*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*a^2*d^4*e*f^3-2100*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^
4*e^3*f+2400*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^4*e^2*f^2-300*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
*b*c^4*d*f^5+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f
+d*e)/(d*f)^(1/2))*b^2*c^4*d*e*f^4+90*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e^2*f^3+150*C*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2
*d^3*e^3*f^2+960*B*x^3*b^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-960*
B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a^2*c*d^4*e*f^4-2400*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^3*f^2-600*B*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^3*f^2-945*C*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b^2*d^5*e^5+640*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c*d^3*f^4-1920*
A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a*b*c*d^4*e*f^4-960*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c^2*d^2*f^
4+1050*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*b^2*d^5*e^4*f-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^3*f^2+3840*A*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*a^2*d^4*f^4+1890*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*
d^4*e^4-1200*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*b^2*d^5*e^3*f^2+1920*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*f^5-1920*A*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e*f^4
-150*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b^2*c^4*d*f^5+240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^3*d^2*f^5-480*B*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^3*f^5+240*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b^2*c^3*d^2*f^5-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^4*f^4)/((d*
x+c)*(f*x+e))^(1/2)/f^5/d^4/(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 43.625, size = 4766, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(63*C*b^2*d^5*e^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)
*e^4*f - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b
 + A*b^2)*d^5)*e^3*f^2 - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8
*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C
*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 1
28*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*
c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x
+ c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b^2*d^5*f^5*x^4 +
945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5)*e^3*f^2 -
 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2 + 2*B*a*b +
 A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B*b^2)*c^2*d^3
+ 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*d^5)*e*f^4 - 1
5*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2
 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f^5 - 48*(9*C*b^2
*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5)*x^3 + 8*(63*C*b^
2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d^5)*e*f^4 - (7*C*b
^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*B*a*b + A*b^2)*d^5)
*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b^2*c*d^4 + 50*(2*C*a*b + B*b
^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*a*b + B*b^2)*c*d^4 - 400*(C*
a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*c^3*d^2 - 10*(2*C*a*b + B*b^
2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 96*(B*a^2 + 2*A*a*b)*d^5)
*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6), 1/3840*(15*(63*C*b^2*d^5*e
^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)*e^4*f - 10*(C*b^2*c^2*d^3 -
 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^3*f^2 - 6*(
C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c
*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C*b^2*c^4*d - 128*A*a^2*d^5 -
 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B
*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b
 + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*
b)*c^2*d^3)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqr
t(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) +
 2*(384*C*b^2*d^5*f^5*x^4 + 945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C
*a*b + B*b^2)*d^5)*e^3*f^2 - 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*
d^4 - 600*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 1
7*(2*C*a*b + B*b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a
^2 + 2*A*a*b)*d^5)*e*f^4 - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b
+ B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a
*b)*c*d^4)*f^5 - 48*(9*C*b^2*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2
)*d^5)*f^5)*x^3 + 8*(63*C*b^2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b
+ B*b^2)*d^5)*e*f^4 - (7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C
*a^2 + 2*B*a*b + A*b^2)*d^5)*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b
```

```
^2*c*d^4 + 50*(2*C*a*b + B*b^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*
a*b + B*b^2)*c*d^4 - 400*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*
c^3*d^2 - 10*(2*C*a*b + B*b^2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4
 + 96*(B*a^2 + 2*A*a*b)*d^5)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.97709, size = 2032, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1920*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8
*(d*x + c)*C*b^2/(d^5*f) - (31*C*b^2*c*d^20*f^8 - 20*C*a*b*d^21*f^8 - 10*B*
b^2*d^21*f^8 + 9*C*b^2*d^21*f^7*e)/(d^25*f^9)) + (263*C*b^2*c^2*d^20*f^8 -
340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*a^2*d^22*f^8 + 160*B*a*b
*d^22*f^8 + 80*A*b^2*d^22*f^8 + 154*C*b^2*c*d^21*f^7*e - 140*C*a*b*d^22*f^7
*e - 70*B*b^2*d^22*f^7*e + 63*C*b^2*d^22*f^6*e^2)/(d^25*f^9)) - 5*(121*C*b^
2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B*b^2*c^2*d^21*f^8 + 112*C*a^
2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^2*c*d^22*f^8 - 96*B*a^2*d^23*
f^8 - 192*A*a*b*d^23*f^8 + 109*C*b^2*c^2*d^21*f^7*e - 200*C*a*b*c*d^22*f^7*
e - 100*B*b^2*c*d^22*f^7*e + 80*C*a^2*d^23*f^7*e + 160*B*a*b*d^23*f^7*e + 8
0*A*b^2*d^23*f^7*e + 91*C*b^2*c*d^22*f^6*e^2 - 140*C*a*b*d^23*f^6*e^2 - 70*
B*b^2*d^23*f^6*e^2 + 63*C*b^2*d^23*f^5*e^3)/(d^25*f^9))*(d*x + c) + 15*(7*C
*b^2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^21*f^8 + 16*C*a^
2*c^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d^22*f^8 - 32*B*a^2*c
*d^23*f^8 - 64*A*a*b*c*d^23*f^8 + 128*A*a^2*d^24*f^8 + 12*C*b^2*c^3*d^21*f^
7*e - 36*C*a*b*c^2*d^22*f^7*e - 18*B*b^2*c^2*d^22*f^7*e + 32*C*a^2*c*d^23*f
^7*e + 64*B*a*b*c*d^23*f^7*e + 32*A*b^2*c*d^23*f^7*e - 96*B*a^2*d^24*f^7*e
- 192*A*a*b*d^24*f^7*e + 18*C*b^2*c^2*d^22*f^6*e^2 - 60*C*a*b*c*d^23*f^6*e^
2 - 30*B*b^2*c*d^23*f^6*e^2 + 80*C*a^2*d^24*f^6*e^2 + 160*B*a*b*d^24*f^6*e^
2 + 80*A*b^2*d^24*f^6*e^2 + 28*C*b^2*c*d^23*f^5*e^3 - 140*C*a*b*d^24*f^5*e^
3 - 70*B*b^2*d^24*f^5*e^3 + 63*C*b^2*d^24*f^4*e^4)/(d^25*f^9))*sqrt(d*x + c
) - 15*(7*C*b^2*c^5*f^5 - 20*C*a*b*c^4*d*f^5 - 10*B*b^2*c^4*d*f^5 + 16*C*a^
2*c^3*d^2*f^5 + 32*B*a*b*c^3*d^2*f^5 + 16*A*b^2*c^3*d^2*f^5 - 32*B*a^2*c^2*
d^3*f^5 - 64*A*a*b*c^2*d^3*f^5 + 128*A*a^2*c*d^4*f^5 + 5*C*b^2*c^4*d*f^4*e
- 16*C*a*b*c^3*d^2*f^4*e - 8*B*b^2*c^3*d^2*f^4*e + 16*C*a^2*c^2*d^3*f^4*e +
 32*B*a*b*c^2*d^3*f^4*e + 16*A*b^2*c^2*d^3*f^4*e - 64*B*a^2*c*d^4*f^4*e - 1
28*A*a*b*c*d^4*f^4*e - 128*A*a^2*d^5*f^4*e + 6*C*b^2*c^3*d^2*f^3*e^2 - 24*C
*a*b*c^2*d^3*f^3*e^2 - 12*B*b^2*c^2*d^3*f^3*e^2 + 48*C*a^2*c*d^4*f^3*e^2 +
96*B*a*b*c*d^4*f^3*e^2 + 48*A*b^2*c*d^4*f^3*e^2 + 96*B*a^2*d^5*f^3*e^2 + 19
```

```
2*A*a*b*d^5*f^3*e^2 + 10*C*b^2*c^2*d^3*f^2*e^3 - 80*C*a*b*c*d^4*f^2*e^3 - 4
0*B*b^2*c*d^4*f^2*e^3 - 80*C*a^2*d^5*f^2*e^3 - 160*B*a*b*d^5*f^2*e^3 - 80*A
*b^2*d^5*f^2*e^3 + 35*C*b^2*c*d^4*f*e^4 + 140*C*a*b*d^5*f*e^4 + 70*B*b^2*d^
5*f*e^4 - 63*C*b^2*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^4*f^5))*d/abs(d)
```

3.48 $\int \frac{(a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=540

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2\left(8\right.\right.}{96bd^3f^3}$$

```
[Out] -((8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3)
+ 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqr
t[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^4) + (C*(a + b*x)^2*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(4*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 +
8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f - 6
*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f +
b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))/(96*b*d^3*f^3) + ((d*e - c*f)*(8*a*d*f
*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))
+ b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2
*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*ArcTanh[(Sqrt
[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(7/2)*f^(9/2))
```

Rubi [A] time = 0.71253, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1615, 147, 50, 63, 217, 206}

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2\left(8\right.\right.}{96bd^3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```


```
[Out] -((8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3)
+ 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqr
t[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^4) + (C*(a + b*x)^2*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(4*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 +
8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f - 6
*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f +
b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))/(96*b*d^3*f^3) + ((d*e - c*f)*(8*a*d*f
*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))
+ b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2
*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*ArcTanh[(Sqrt
[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(7/2)*f^(9/2))
```


Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
```

```
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}dx = \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)\sqrt{c+dx}\left(-\frac{1}{2}b(4bcCe+3aCde+acCf-8Abdf)-\frac{1}{2}b(4aC}{\sqrt{e+fx}}}{4b^2df}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} - \frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+8abdf(5Cde\right.}{}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+b\left(C\left(35d^3e^3\right.\right.\right.}{}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+b\left(C\left(35d^3e^3\right.\right.\right.}{}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+b\left(C\left(35d^3e^3\right.\right.\right.}{}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+b\left(C\left(35d^3e^3\right.\right.\right.}{}$$

Mathematica [B] time = 6.32651, size = 2402, normalized size = 4.45

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((-(b*e) + a*f)*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^
2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c +
d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2*d^3*f^5*Sqrt[c + d*x]*Sqrt[e + f
*x]) + (2*b*C*(d*e - c*f)^2*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*
x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5
/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^3) + 5/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcS
inh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f)))])))/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^
3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/
(d*e - c*f)]) + (2*(d*e - c*f)*(-3*b*C*e + b*B*f + a*C*f)*(c + d*x)^(3/2)*S
qrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(
```

```
d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((
d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c +
 d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)
/(d*e - c*f) - (c*d*f)/(d*e - c*f)]])]/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)
/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*
(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/
(3*d^2*f^3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e
 + f*x))/(d*e - c*f)]) + (2*(3*b*C*e^2 - 2*b*B*e*f - 2*a*C*e*f + A*b*f^2 +
a*B*f^2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((
d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c
*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*
Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt
[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]])]/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]])))/(16*d^2*f^2*(c + d*x)^2*(1
 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))
)))))/(3*d*f^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*
(e + f*x))/(d*e - c*f)])
```

Maple [B] time = 0.026, size = 2002, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*f^4+105*C*ln(1/2*(2*d*f*x+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^4-60*C*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b*c*d^3*e^3*f-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e*f^2-288*B*
(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*e*f^2+240*B*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*b*d^3*e^2*f+72*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e^2*f^2+24*C*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*e*f^3-12*C
*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*b*c^3*d*e*f^3-18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e^2*f^2-96*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e*f^3+50*C*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*e^2*f+32*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*x*a*c*d^2*f^3-160*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*d^3*e*f^2
-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*c^2*d*f^3+140*C*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*x*b*d^3*e^2*f+32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*x*b*c*d^2*f^3-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^3*e*f^2-64
*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*e*f^2+16*C*x^2*b*c*d^2*f^3*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-112*C*x^2*b*d^3*e*f^2*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)-64*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*e*f^2+34
*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d*e*f^2-24*C*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*x*b*c*d^2*e*f^2-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*f^4+96*C*x^3*b*d^3*f^3*(d*f)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)+128*B*x^2*b*d^3*f^3*(d*f)^(1/2)*((d*x+c)*(f*
```

```
x+e))^(1/2)+128*C*x^2*a*d^3*f^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-48*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d*f^3+192*B*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*x*a*d^3*f^3+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e*f^3+96*A*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b*c*d^2*f^3+96*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*f
^3-48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d*f^3-96*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e*f^3
+72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*b*c*d^3*e^2*f^2+240*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*e^2*
f+192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^3*f^3+192*A*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*f^4-
192*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*a*d^4*e*f^3+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^2*f^2+144*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^2*f^2-120*B*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e
^3*f-120*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*a*d^4*e^3*f+384*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*f^3
-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e^3+30*C*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b*c^3*f^3-48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*f^4-48*B*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*f^4+24*C*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*c^3*d*f^4)/f^4/((d*x+c)*(f*x+e))^(1/2)/d^3/(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.36583, size = 2503, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/768*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C
*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*
d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*
b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*s
qrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d
*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x)
+ 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^3 + 24*(C*a + B*
b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a + A*b)*
d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 + 16*(B*a
+ A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*a + B*b)*d^4)*f^
```

```
4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a + B*b)*d^4)*e*f^3
 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^4)*f^4)*x)*sqrt(
d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^
3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*
a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 +
 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3
*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*
f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f
 + c*d*f^2)*x)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^
3 + 24*(C*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 -
144*(B*a + A*b)*d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^
2*d^2 + 16*(B*a + A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*
a + B*b)*d^4)*f^4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a +
 B*b)*d^4)*e*f^3 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^
4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39128, size = 994, normalized size = 1.84

$$\left(\sqrt{(dx+c)df-cdf+d^2e}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb}{d^4f}-\frac{17\,Cbcd^{12}f^6-8\,Cad^{13}f^6-8\,Bbd^{13}f^6+7\,Cbd^{13}f^5e}{d^{16}f^7}\right)+\frac{59\,Cbc^2d^{12}f^6-56\,Cac\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b/(d^4*f) - (17*C*b*c*d^12*f^6 - 8*C*a*d^13*f^6 - 8*B*b*d^13*f^6 +
 7*C*b*d^13*f^5*e)/(d^16*f^7)) + (59*C*b*c^2*d^12*f^6 - 56*C*a*c*d^13*f^6 -
 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6 + 50*C*b*c*d^13*f^5*
e - 40*C*a*d^14*f^5*e - 40*B*b*d^14*f^5*e + 35*C*b*d^14*f^4*e^2)/(d^16*f^7)
) - 3*(5*C*b*c^3*d^12*f^6 - 8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*
a*c*d^14*f^6 + 16*A*b*c*d^14*f^6 - 64*A*a*d^15*f^6 + 9*C*b*c^2*d^13*f^5*e -
 16*C*a*c*d^14*f^5*e - 16*B*b*c*d^14*f^5*e + 48*B*a*d^15*f^5*e + 48*A*b*d^1
5*f^5*e + 15*C*b*c*d^14*f^4*e^2 - 40*C*a*d^15*f^4*e^2 - 40*B*b*d^15*f^4*e^2
 + 35*C*b*d^15*f^3*e^3)/(d^16*f^7))*sqrt(d*x + c) + 3*(5*C*b*c^4*f^4 - 8*C*
a*c^3*d*f^4 - 8*B*b*c^3*d*f^4 + 16*B*a*c^2*d^2*f^4 + 16*A*b*c^2*d^2*f^4 - 6
4*A*a*c*d^3*f^4 + 4*C*b*c^3*d*f^3*e - 8*C*a*c^2*d^2*f^3*e - 8*B*b*c^2*d^2*f
^3*e + 32*B*a*c*d^3*f^3*e + 32*A*b*c*d^3*f^3*e + 64*A*a*d^4*f^3*e + 6*C*b*c
^2*d^2*f^2*e^2 - 24*C*a*c*d^3*f^2*e^2 - 24*B*b*c*d^3*f^2*e^2 - 48*B*a*d^4*f
^2*e^2 - 48*A*b*d^4*f^2*e^2 + 20*C*b*c*d^3*f*e^3 + 40*C*a*d^4*f*e^3 + 40*B*
b*d^4*f*e^3 - 35*C*b*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

3.49 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8d^2f^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2df(4Adf-B(\right.}{8d^{5/2}f^{}}$$

```
[Out] ((C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(8*d^2*f^3) - ((5*C*d*e + 7*c*C*f - 6*B*d*f)*(
c + d*x)^(3/2)*Sqrt[e + f*x])/(12*d^2*f^2) + (C*(c + d*x)^(5/2)*Sqrt[e + f*
x])/(3*d^2*f) - ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(
4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
 + f*x])])/(8*d^(5/2)*f^(7/2))
```

Rubi [A] time = 0.230315, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8d^2f^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2df(4Adf-B(\right.}{8d^{5/2}f^{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(8*d^2*f^3) - ((5*C*d*e + 7*c*C*f - 6*B*d*f)*(
c + d*x)^(3/2)*Sqrt[e + f*x])/(12*d^2*f^2) + (C*(c + d*x)^(5/2)*Sqrt[e + f*
x])/(3*d^2*f) - ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(
4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
 + f*x])])/(8*d^(5/2)*f^(7/2))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}+\frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}\left(-5cCde-c^2Cf+6Ad^2f\right)-\frac{1}{2}d(5Cde+7cCf-6Bdf)x\right)}{\sqrt{e+fx}}\,dx}{3d^2f}$$

$$= -\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}+\frac{\left(C\left(5d^2e^2+2\right.\right.}{}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}-\frac{(5Cde}{}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}-\frac{(5Cde}{}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}-\frac{(5Cde}{}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}-\frac{(5Cde}{}$$

Mathematica [A] time = 1.06982, size = 225, normalized size = 0.91

$$\frac{-d\sqrt{f}\sqrt{c+dx}(e+fx)\left(C\left(3c^2f^2-2cdf(fx-2e)+d^2\left(-15e^2+10efx-8f^2x^2\right)\right)-6df(4Adf+B(cf-3de+2dfx))\right.}{24d^3f^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-(d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(-6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2
*d*f*x)) + C*(3*c^2*f^2 - 2*c*d*f*(-2*e + f*x) + d^2*(-15*e^2 + 10*e*f*x -
8*f^2*x^2)))) - 3*(d*e - c*f)^(3/2)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[
(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(24*d^3*f^(7/2)*Sqrt[e + f*x])
```

Maple [B] time = 0.018, size = 763, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*x^2*d^2*f^2*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)+24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e*f^
2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*d^3*e^2*f+24*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d^2*f^2+3*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c
^3*f^3+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e^3+4*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*x*c*d*f^2-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d^
2*e*f+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2+12*B*(d*f)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e
*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*f^2-8*C*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e^2)/f
^3/((d*x+c)*(f*x+e))^(1/2)/d^2/(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5108, size = 1277, normalized size = 5.19

$$\left[-\frac{3\left(5\,Cd^3e^3-3\left(Ccd^2+2\,Bd^3\right)e^2f-\left(Cc^2d-4\,Bcd^2-8\,Ad^3\right)ef^2-\left(Cc^3-2\,Bc^2d+8\,Acd^2\right)f^3\right)\sqrt{df}\log\left(8\,d^2f^2x^2+d^2e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^2
 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*d^3*f^3*
x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*d - 2*B*c*d
^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*x)*sqrt(d*x
 + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3
)*e^2*f - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*
c*d^2)*f^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x
 + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8
*C*d^3*f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*
d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*
x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.66155, size = 425, normalized size = 1.73

$$\left(\sqrt{(dx+c)df - cdf + d^2e}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f} - \frac{7\,Ccd^6f^4-6\,Bd^7f^4+5\,Cd^7f^3e}{d^9f^5}\right) + \frac{3\left(Cc^2d^6f^4-2\,Bcd^7f^4+8\,Ad^8f^4+2\,Ccd^7f^3e-6\right.}{d^9f^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C/(d^3*f) - (7*C*c*d^6*f^4 - 6*B*d^7*f^4 + 5*C*d^7*f^3*e)/(d^9*f^5))
 + 3*(C*c^2*d^6*f^4 - 2*B*c*d^7*f^4 + 8*A*d^8*f^4 + 2*C*c*d^7*f^3*e - 6*B*d
^8*f^3*e + 5*C*d^8*f^2*e^2)/(d^9*f^5)) - 3*(C*c^3*f^3 - 2*B*c^2*d*f^3 + 8*A
*c*d^2*f^3 + C*c^2*d*f^2*e - 4*B*c*d^2*f^2*e - 8*A*d^3*f^2*e + 3*C*c*d^2*f*
e^2 + 6*B*d^3*f*e^2 - 5*C*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

3.50 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal. Leaf size=290

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{4b^3d^{3/2}f^{5/2}}-\frac{2\sqrt{bc-ad}}{}$$

```
[Out] -((4*a*C*d*f + b*(3*C*d*e + c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + ((2*b*d*f*(4*
A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(3*
C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(4*b^3*d^(3/2)*f^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])
])/(b^3*Sqrt[b*e - a*f])
```

Rubi [A] time = 0.672355, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1615, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{4b^3d^{3/2}f^{5/2}}-\frac{2\sqrt{bc-ad}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```


```
[Out] -((4*a*C*d*f + b*(3*C*d*e + c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + ((2*b*d*f*(4*
A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(3*
C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(4*b^3*d^(3/2)*f^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])
])/(b^3*Sqrt[b*e - a*f])
```


Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}b(4Abdf-aC(3de+cf))-\frac{1}{2}b(4aCdf+b(3Cde+cCf-4Bdf))x\right)}{(a+bx)\sqrt{e+fx}}dx}{2b^2df}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{1}{4}b(2b \ldots}{\ldots}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\left(\left(Ab^2 \ldots\right.\right.}{\ldots}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\left(2\left(Ab^2 \ldots\right.\right.}{\ldots}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} - \frac{2\left(Ab^2 \ldots\right.}{\ldots}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{(2bdf(\ldots}{\ldots}$$

Mathematica [A] time = 3.85997, size = 465, normalized size = 1.6

$$\frac{-\frac{8\sqrt{ad-bc}\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{be-af}} + \frac{8\sqrt{de-cf}\left(a(aC-bB)+Ab^2\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{4b\sqrt{e+fx}(aCf-bBf+bCe)\left(\sqrt{c+dx}(de-cf)\sinh \ldots\right.}{f^{5/2}\sqrt{c+dx}\sqrt{de- \ldots}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] ((8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (4*b*(b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*(-(Sqrt[f]*Sqrt[d*e - c*f]*
(c + d*x)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (d*e - c*f)*Sqrt[c + d*x]*ArcS
inh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(f^(5/2)*Sqrt[d*e - c*f]*Sqr
t[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (b^2*C*Sqrt[e + f*x]*(Sqrt[f]
*Sqrt[c + d*x]*(c*f + d*(e + 2*f*x)) - ((d*e - c*f)^(3/2)*ArcSinh[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)]))/(d*f^(5/
2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b*e - a
*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[b*e - a*f])/(4
*b^3)
```

Maple [B] time = 0.033, size = 1822, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x)
```

```
[Out] 1/8*(8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^2*
d^2*f^2*(d*f)^(1/2)-8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*b^3*c*d*f^2*(d*f)^(1/2)+8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)-8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*a^2*b*d^2*f^2*(d*f)^(1/2)+8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*a*b^2*c*d*f^2*(d*f)^(1/2)-8*B*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^2*d^2*f^2*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*c*d*f^2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)+8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a^3*d^2*f^2*(d*f)^(1/2)-8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e
+2*b*c*e)/(b*x+a))*a^2*b*c*d*f^2*(d*f)^(1/2)+8*C*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b*d^2*f^2*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^2*c*d*f^2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^2*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^3*c^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*b^3*c*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^
2*e^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*C*x*b^3*d*f*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*
B*b^3*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)-8*C*a*b^2*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^3*c*f*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-6*C*b^3*d*e*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
/(d*f)^(1/2)/d/f^2/b^4/((d*x+c)*(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

Giac [B] time = 1.80923, size = 797, normalized size = 2.75

$$\frac{1}{4}\sqrt{(dx+c)df-cdf+d^2e}\sqrt{dx+c}\left(\frac{2\,(dx+c)C}{bdf|d|}-\frac{Cb^5cd^3f^2+4\,Cab^4d^4f^2-4\,Bb^5d^4f^2+3\,Cb^5d^4fe}{b^6d^4f^3|d|}\right)-\frac{2\left(\sqrt{df}Ca^2bcd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/4*sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C/(b*d*f
*abs(d)) - (C*b^5*c*d^3*f^2 + 4*C*a*b^4*d^4*f^2 - 4*B*b^5*d^4*f^2 + 3*C*b^5
*d^4*f*e)/(b^6*d^4*f^3*abs(d))) - 2*(sqrt(d*f)*C*a^2*b*c*d^2 - sqrt(d*f)*B*
a*b^2*c*d^2 + sqrt(d*f)*A*b^3*c*d^2 - sqrt(d*f)*C*a^3*d^3 + sqrt(d*f)*B*a^2
*b*d^3 - sqrt(d*f)*A*a*b^2*d^3)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e
- (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqr
t(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/(sqrt(a*b*c*d*
f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*b^3*d*abs(d)) + 1/8*(sqrt(d*
f)*C*b^2*c^2*f^2 + 4*sqrt(d*f)*C*a*b*c*d*f^2 - 4*sqrt(d*f)*B*b^2*c*d*f^2 -
8*sqrt(d*f)*C*a^2*d^2*f^2 + 8*sqrt(d*f)*B*a*b*d^2*f^2 - 8*sqrt(d*f)*A*b^2*d
^2*f^2 + 2*sqrt(d*f)*C*b^2*c*d*f*e - 4*sqrt(d*f)*C*a*b*d^2*f*e + 4*sqrt(d*f
)*B*b^2*d^2*f*e - 3*sqrt(d*f)*C*b^2*d^2*e^2)*log((sqrt(d*f)*sqrt(d*x + c) -
 sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2)/(b^3*d*f^3*abs(d))
```

3.51 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal. Leaf size=364

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)}{b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-a^2b(2Bdf+3cCf+\right.}{}$$

```
[Out] ((2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[c +
 d*x]*Sqrt[e + f*x])/(b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 - a*(b*B - a
*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e - a*f)*(a + b*x)) -
 ((4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqrt[d]*f^(3/2)) + ((4*a^3*C*d*f - b^3*(2*B
*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^2*b*(5*C*d*e
+ 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a
*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*c - a*d]*(b*e - a*f)^(3/2))
```

Rubi [A] time = 1.09748, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1613, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)}{b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-a^2b(2Bdf+3cCf+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```


```
[Out] ((2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[c +
 d*x]*Sqrt[e + f*x])/(b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 - a*(b*B - a
*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e - a*f)*(a + b*x)) -
 ((4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqrt[d]*f^(3/2)) + ((4*a^3*C*d*f - b^3*(2*B
*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^2*b*(5*C*d*e
+ 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a
*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*c - a*d]*(b*e - a*f)^(3/2))
```


Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(2Bce+Ade-Acf)-ab(2cCe+3B\cdots}{2b}\cdots\right)}{(\cdots}}{(bc-\cdots}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(Ab^2-a(b\cdots\right.}{b(bc-\cdots}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(Ab^2-a(b\cdots\right.}{b(bc-\cdots}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(Ab^2-a(b\cdots\right.}{b(bc-\cdots}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(Ab^2-a(b\cdots\right.}{b(bc-\cdots}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(Ab^2-a(b\cdots\right.}{b(bc-\cdots}$$

Mathematica [A] time = 2.92368, size = 417, normalized size = 1.15

$$\frac{-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)(be-af)}+\frac{2b(de-cf)\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}(be-af)^{3/2}}-\frac{4(bB-2aC)\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{be-af}}+\frac{4(bB-2aC)\sqrt{de-cf}\cdots}{\cdots}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)
*(a + b*x)) + (4*(b*B - 2*a*C)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (2*b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x] - (Sqrt[d*e - c*f]*ArcSinh
[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)])
)/f^(3/2) - (4*(b*B - 2*a*C)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b*e - a*f]*Sqr
t[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[b*e - a*f] + (2*b*(A*
b^2 + a*(-(b*B) + a*C))*(d*e - c*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/
(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(b*e - a*f)^(3/2))
)/(2*b^3)
```

Maple [B] time = 0.042, size = 3670, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a
*d*e+2*b*c*e)/(b*x+a))*a^4*d*f^2*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*c*e*f*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*a^2*b^2*d*
f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*a*b^3*c*f^2*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*b^4*c*e*f*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*x*a^3*b*d*f^2*(d*f)^(1/2)-3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*
c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a^2*b^2*c*f^2*(d*f)^(1/2)+C*ln(1/2*(2*d*f*x+2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*e*f*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*
x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*f*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*e*f*(d*f)^(1/2)-2*A*b
^4*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/b^2)^(1/2)+3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*x*a*b^3*d*e*f*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*a*b^3*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*x*a^2*b^2*d*e*f*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a
*d*e+2*b*c*e)/(b*x+a))*x*a*b^3*c*e*f*(d*f)^(1/2)+2*B*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e*f*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*f*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*f*(d*f)^(1/2)-3*C*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b
^2*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*x*a*b^3*f*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*d*e*f*
(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*x*a*b^3*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)+2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*x*b^4*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*B*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a^2*b^2*d*f^2*(d*f)
^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a*b^
3*c*f^2*(d*f)^(1/2)+2*B*a*b^3*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4*C*a^2*b^2*f*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*a*b^3*e*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*c*f
^2*(d*f)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*x*b^4*d*e^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
```

```
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f^2*
(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*a^2*b^2*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*f
^2*(d*f)^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a)
)*a^2*b^2*c*f^2*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f
+d*e)/(d*f)^(1/2))*a^2*b^2*c*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*a*b^3*d*e^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-3*C*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f^2*(d*f)^(1/
2)+2*C*x*b^4*e*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*a*b^3*d*e*f*(d*f)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/(a*f-b*e)/f/(d*f)
^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/(b*x+a)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 12.5035, size = 1874, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 4*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*B*a^2*b*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 5*sqrt(d*f)*C*a^2*b*d^3*e -
3*sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2
*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)
*d))/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*(a*b^3*f*
abs(d) - b^4*abs(d)*e)*d) + 2*(sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*
a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4
*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*
b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
 + d^2*e))^2*A*a*b^2*d^3*f + sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*
d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^
3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)*(a*b^3*f*abs(d) -
 b^4*abs(d)*e)) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*C*abs(d
)/(b^2*d^2*f) - 1/2*(sqrt(d*f)*C*b*c*f - 4*sqrt(d*f)*C*a*d*f + 2*sqrt(d*f)*
B*b*d*f - sqrt(d*f)*C*b*d*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2)/(b^3*f^2*abs(d))
```

3.52 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal. Leaf size=484

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(3a^2b^2C\left(c^2f^2+10cdef+5d^2e^2\right)-4a^3bCdf(3cf+5de)+8a^4Cd^2f^2-ab^3\left(2cd\left(2Af^2-Bef\right.\right.\right.}{4b^3(bc-ad)^{3/2}(}$$

```
[Out] ((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^3*(4*B*c*e - A*d*e - 3*A*c*f) +
 a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c +
 d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (2*C
*Sqrt[d]*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqr
t[f]) - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*
d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e - 4*A*f) + c^2*f*(8*C
*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^4*(A*d^2*e^2 - 2*c*d*e*
(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^3*(b*c - a*d)^(3/2)
*(b*e - a*f)^(5/2))
```

Rubi [A] time = 1.56326, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1613, 149, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(3a^2b^2C\left(c^2f^2+10cdef+5d^2e^2\right)-4a^3bCdf(3cf+5de)+8a^4Cd^2f^2-ab^3\left(2cd\left(2Af^2-Bef\right.\right.\right.}{4b^3(bc-ad)^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```


```
[Out] ((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^3*(4*B*c*e - A*d*e - 3*A*c*f) +
 a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c +
 d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (2*C
*Sqrt[d]*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqr
t[f]) - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*
d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e - 4*A*f) + c^2*f*(8*C
*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^4*(A*d^2*e^2 - 2*c*d*e*
(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^3*(b*c - a*d)^(3/2)
*(b*e - a*f)^(5/2))
```


Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
```

```
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(4Bce-Ade-3Acf)-ab(4cCe+3B}{2b}\right.}{(a+bx)^2\sqrt{e+fx}}}{2(bc-ad)(be-}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

Mathematica [A] time = 6.30474, size = 535, normalized size = 1.11

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)} + \frac{\left(Ab^2-a(bB-aC)\right)(-4adf+3bcf+bde)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{(a+bx)(be-af)} - \frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right.}{\sqrt{ad-bc}(be-af)}\right.}{4b^2(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] -(((b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])/(b^2*(b*e - a*f)*(a + b*x)))
- ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^2) + (2*C*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c
*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(b^3*Sqrt[f]*Sqrt[e
+ f*x]) - (2*C*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(S
qrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*e - a*f]) + ((b*B - 2*a*C)*(
d*e - c*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[
e + f*x])])/(b^2*Sqrt[-(b*c) + a*d]*(b*e - a*f)^(3/2)) + ((A*b^2 - a*(b*B -
 a*C))*(b*d*e + 3*b*c*f - 4*a*d*f)*((Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a
*f)*(a + b*x)) - ((d*e - c*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(b*e - a*f)^(3/2))))/(4*
b^2*(b*c - a*d)*(b*e - a*f))
```

Maple [B] time = 0.088, size = 9100, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.53 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal. Leaf size=685

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-a^2b^2\left(4df(-2Adf+Bcf+4Bde)-C\left(3c^2f^2+44cdef+33d^2e^2\right)\right)-2a^3bdf(-2Bdf+7cCf+13\ldots}{\ldots}$$

```
[Out] ((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A*c*f) + a*b^2*(12*c*C*e + 3*B*d
*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*c*C*f - 2*B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(12*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) - ((8*a^4*C*d
^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4*(3*A*d^2*e^2 - 2*
c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C*e^2 - 14*B*e*f + 13
*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f) - C*(33*d^2*e^2 + 44*
c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b^2*(b*c - a*d)^2*(
b*e - a*f)^3*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e +
 f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - ((d*e - c*f)*(b^2*(A*d^2
*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2*
e*(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^2)
) - a^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f
*x])])/(8*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2))
```

Rubi [A] time = 1.77788, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1613, 149, 151, 12, 93, 208}

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-a^2b^2\left(4df(-2Adf+Bcf+4Bde)-C\left(3c^2f^2+44cdef+33d^2e^2\right)\right)-2a^3bdf(-2Bdf+7cCf+13\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A*c*f) + a*b^2*(12*c*C*e + 3*B*d
*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*c*C*f - 2*B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(12*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) - ((8*a^4*C*d
^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4*(3*A*d^2*e^2 - 2*
c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C*e^2 - 14*B*e*f + 13
*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f) - C*(33*d^2*e^2 + 44*
c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b^2*(b*c - a*d)^2*(
b*e - a*f)^3*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e +
 f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - ((d*e - c*f)*(b^2*(A*d^2
*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2*
e*(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^2)
) - a^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f
*x])])/(8*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2))
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
```

```
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(6Bce-3Ade-5Acf)-ab(6cCe+3\cdots}{2b}\cdots\right.}{\cdots}}{3(b\cdots}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9C\cdots\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9C\cdots\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9C\cdots\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9C\cdots\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9C\cdots\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

Mathematica [A] time = 6.33094, size = 739, normalized size = 1.08

$$-\frac{\left(a^2C-abB+Ab^2\right)\left(-\frac{3\left(8a^2d^2f^2-4abdf(3cf+de)+b^2\left(5c^2f^2+2cdef+d^2e^2\right)\right)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{(a+bx)(af-be)}-\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\sqrt{be-af}(af-be)}\right)}{8(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(\frac{1}{2}b(-6\cdots\right.}{2(a+bx)^2(bc\cdots}\cdots\right.}{3b^2(bc-ad)(be-af)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] -((C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b^2*(b*e - a*f)*(a + b*x))) - ((A*b^2 -
a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*
(a + b*x)^3) - ((b*B - 2*a*C)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*
d)*(b*e - a*f)*(a + b*x)^2) + (C*(d*e - c*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[-(b*c) + a*d]*(b*e
- a*f)^(3/2)) + ((b*B - 2*a*C)*(b*d*e + 3*b*c*f - 4*a*d*f)*((Sqrt[c + d*x]*
Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) - ((d*e - c*f)*ArcTan[(Sqrt[b*e - a*
f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*
(b*e - a*f)^(3/2))))/(4*b^2*(b*c - a*d)*(b*e - a*f)) - ((A*b^2 - a*b*B + a^
2*C)*(-((-(a*b*d*f) + (b*(3*b*d*e + 5*b*c*f - 6*a*d*f))/2)*(c + d*x)^(3/2)*
Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (3*(8*a^2*d^2*f^2
- 4*a*b*d*f*(d*e + 3*c*f) + b^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*((Sqrt[c
 + d*x]*Sqrt[e + f*x])/((-(b*e) + a*f)*(a + b*x)) - ((d*e - c*f)*ArcTan[(Sq
rt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b
*c) + a*d]*Sqrt[b*e - a*f]*(-(b*e) + a*f))))/(8*(b*c - a*d)*(b*e - a*f))))/
(3*b^2*(b*c - a*d)*(b*e - a*f))
```

Maple [B] time = 0.148, size = 15990, normalized size = 23.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.54 $\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=718

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-8a^2bd^2f^2(16Bdf-11C(cf+de))+32a^3Cd^3f^3-16ab^2df\left(6df(4Adf-3B(cf+de))+C\left(15c^2f\right.\right.\right.}{}$$

```
[Out] -((2*a*C*d*f - b*(8*B*d*f - 7*C*(d*e + c*f)))*(a + b*x)^2*Sqrt[c + d*x]*Sqr
t[e + f*x])/(24*b*d^2*f^2) + (C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4
*b*d*f) - (Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*
(16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f +
15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 + 1
9*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*f)
- B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*C*e
+ a*C*d*e + a*c*C*f - 8*A*b*d*f) + (4*a*d*f - 5*b*(d*e + c*f))*(2*a*C*d*f -
 b*(8*B*d*f - 7*C*(d*e + c*f))))*x))/(192*b*d^4*f^4) + ((16*a^2*d^2*f^2*(C*
(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) - 16
*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f
*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) + b^2*(C*(3
5*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f
^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3
*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x]
)/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(9/2)*f^(9/2))
```

Rubi [A] time = 1.33552, antiderivative size = 715, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1615, 153, 147, 63, 217, 206}

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-8a^2bd^2f^2(16Bdf-11C(cf+de))+32a^3Cd^3f^3-16ab^2df\left(6df(4Adf-3B(cf+de))+C\left(15c^2f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((8*b*B*d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt
[e + f*x])/(24*b*d^2*f^2) + (C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*
b*d*f) - (Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(
16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f + 1
5*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 + 19
*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*f) -
 B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*C*e +
 a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*d*f -
2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(192*b*d^4*f^4) + ((16*a^2*d^2*f^2*(C*(
3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) - 16*
a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*
(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) + b^2*(C*(35
*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^
4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*
c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(9/2)*f^(9/2))
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)^2\left(-\frac{1}{2}b(6bcCe+aCde+acCf-8Abdf)+\frac{1}{2}b(8bBdf-2aCdf-7b}{\sqrt{c+dx}\sqrt{e+fx}}}{4b^2df}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}}{4bdf}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}}{4bdf}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}}{4bdf}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}}{4bdf}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}}{4bdf}$$

Mathematica [B] time = 6.51798, size = 2195, normalized size = 3.06

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(b*e - a*f)^2*Sqrt[d*e - c*f]*(C*e^2 - f*(B*e - A*f))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(d*f^(9/2)*
Sqrt[e + f*x]) + (2*b^2*C*(d*e - c*f)^3*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9
/2)*((35/(16*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))^4) + 35/(24*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(6*(1 + (d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/8 + (35*S
qrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqr
t[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f)])])/(128*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2))))/(d^4*
f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(d*(e + f*x)
)/(d*e - c*f)]) + (2*b*(d*e - c*f)^2*(-4*b*C*e + b*B*f + 2*a*C*f)*Sqrt[c +
d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^(7/2)*((15/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(4*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c
 + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/6
 + (5*Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSi
nh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)])])/(16*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)))
/(d^3*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]) + (2*(d*e - c*f)*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6*a*b*C
```

```
*e*f + A*b^2*f^2 + 2*a*b*B*f^2 + a^2*C*f^2)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
)^(5/2)*((3/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f))))^(-1))/4 + (3*Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sq
rt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]])/(8*Sqrt[d]
*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f))))^(5/2))))/(d^2*f^4*(d/((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*
f)*(4*b*C*e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))))^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[
d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]])/(2*Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f))))^(3/2))))/(d*f^4*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)])
```

Maple [B] time = 0.04, size = 2528, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/384*(384*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*a^2*d^4*f^4-320*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c
*d^2*f^3-320*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*d^3*e*f^2+136*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d^2*e*f^2+448*C*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*a*b*c*d^2*e*f^2-192*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^4*e*f^3+96*A*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e*f
^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f
)^(1/2))*b^2*c^2*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+96*C*ln(1/2*(2*d*f*x+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^3*e*f^3+60
*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*b^2*c^3*d*e*f^3+54*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^2*e^2*f^2+60*C*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^3*f+192*C*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a^2*d^3*f^3+240*B*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*b^2*d^3*e^2*f+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^4-576*B*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^2*f^3-576*B*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*a*b*d^3*e*f^2+192*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e*f^3-144*C*ln(1/2*(2*d*f*x+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^2*e*f
^3-144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*a*b*c*d^3*e^2*f^2-112*C*x^2*b^2*d^3*e*f^2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+256*C*x^2*a*b*d^3*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+224*
B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^2*e*f^2+480*C*(d*f)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*a*b*c^2*d*f^3+480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*a*b*d^3*e^2*f-190*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d*e*f^2-1
```

```
90*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^2*e^2*f-112*C*x^2*b^2*c*d^
2*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*x*a*b*d^3*f^3-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d^
2*f^3-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^3*e*f^2+140*C*(d*f)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^2*d*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(
f*x+e))^(1/2)*x*b^2*d^3*e^2*f-288*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2
*c*d^2*f^3-288*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^3*e*f^2+96*C*x^3
*b^2*d^3*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+128*B*x^2*b^2*d^3*f^3*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x
*b^2*d^3*f^3-384*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a*b*c*d^3*f^4-384*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e*f^3+288*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^2*f
^4+288*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*a*b*d^4*e^2*f^2-240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*d*f^4-240*C*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e^3*f+768*A*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(
f*x+e))^(1/2)*b^2*c*d^2*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d
^3*e*f^2+240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d*f^3-120*B*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2
*c^3*d*f^4-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^2*d^4*e^3*f+144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^2*f^4+144*C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^4*e^2*f
^2+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^3*f^3-210*C*(d*f)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b^2*c^3*f^3-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*b^2*d^3*e^3+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^2*f^4+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^2*f^2-192*B*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d
^3*f^4)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2)/f^4/d^4/((d*x+c)*(f*x+e))^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 14.8659, size = 3170, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/768*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^
3*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b +
 A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(
C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^
2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b +
A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x
^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt
(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*b^2*d^4*f^4*x^
3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2
*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a
*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 +
 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*
C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(3
5*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 +
 (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b
^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1/384*(3*(35*C*b^
2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^3*f + 6*(3*C*b^2*c
^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e^2*f
^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(C*a^2 + 2*B*a*b +
A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^2*c^4 + 128*A*a^2*
d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - 6
4*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)
*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f +
 c*d*f^2)*x)) - 2*(48*C*b^2*d^4*f^4*x^3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2
*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a
*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*
c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 -
 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*
(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(35*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d
^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 + (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*
b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f
*x + e))/(d^5*f^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.7297, size = 1284, normalized size = 1.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b^2/(d^5*f) - (25*C*b^2*c*d^19*f^6 - 16*C*a*b*d^20*f^6 - 8*B*b^2*d
```

```
^20*f^6 + 7*C*b^2*d^20*f^5*e)/(d^24*f^7)) + (163*C*b^2*c^2*d^19*f^6 - 208*C
*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^21*f^6 + 96*B*a*b*d^21*
f^6 + 48*A*b^2*d^21*f^6 + 90*C*b^2*c*d^20*f^5*e - 80*C*a*b*d^21*f^5*e - 40*
B*b^2*d^21*f^5*e + 35*C*b^2*d^21*f^4*e^2)/(d^24*f^7)) - 3*(93*C*b^2*c^3*d^1
9*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*f^6 + 80*C*a^2*c*d^21*f^
6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 64*B*a^2*d^22*f^6 - 128*A*
a*b*d^22*f^6 + 73*C*b^2*c^2*d^20*f^5*e - 128*C*a*b*c*d^21*f^5*e - 64*B*b^2*
c*d^21*f^5*e + 48*C*a^2*d^22*f^5*e + 96*B*a*b*d^22*f^5*e + 48*A*b^2*d^22*f^
5*e + 55*C*b^2*c*d^21*f^4*e^2 - 80*C*a*b*d^22*f^4*e^2 - 40*B*b^2*d^22*f^4*e
^2 + 35*C*b^2*d^22*f^3*e^3)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C*b^2*c^4*f^4
 - 80*C*a*b*c^3*d*f^4 - 40*B*b^2*c^3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*
b*c^2*d^2*f^4 + 48*A*b^2*c^2*d^2*f^4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3
*f^4 + 128*A*a^2*d^4*f^4 + 20*C*b^2*c^3*d*f^3*e - 48*C*a*b*c^2*d^2*f^3*e -
24*B*b^2*c^2*d^2*f^3*e + 32*C*a^2*c*d^3*f^3*e + 64*B*a*b*c*d^3*f^3*e + 32*A
*b^2*c*d^3*f^3*e - 64*B*a^2*d^4*f^3*e - 128*A*a*b*d^4*f^3*e + 18*C*b^2*c^2*
d^2*f^2*e^2 - 48*C*a*b*c*d^3*f^2*e^2 - 24*B*b^2*c*d^3*f^2*e^2 + 48*C*a^2*d^
4*f^2*e^2 + 96*B*a*b*d^4*f^2*e^2 + 48*A*b^2*d^4*f^2*e^2 + 20*C*b^2*c*d^3*f*
e^3 - 80*C*a*b*d^4*f*e^3 - 40*B*b^2*d^4*f*e^3 + 35*C*b^2*d^4*e^4)*log(abs(-
sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*
d^4*f^4))*d/abs(d)
```

3.55 $\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2+2bdfx(2aCdf-b(6Bdf-5C(cf+de)))-6abdf(4Bdf-3C(cf+de))+b^2\left(-\left(6df(4A\right.\right.\right.}{24bd^3f^3}$$

```
[Out] (C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (Sqrt[c + d*x]*Sqrt
[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2*(C
*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))
) + 2*b*d*f*(2*a*C*d*f - b*(6*B*d*f - 5*C*(d*e + c*f)))*x))/(24*b*d^3*f^3)
+ ((2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*
e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) +
 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcT
anh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*d^(7/2)*f^(7/2))
```

Rubi [A] time = 0.5094, antiderivative size = 369, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1615, 147, 63, 217, 206}

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))+b^2\left(-\left(6df(4A\right.\right.\right.}{24bd^3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (Sqrt[c + d*x]*Sqrt
[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2*(C
*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))
) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(24*b*d^3*f^3)
+ ((2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*
e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) +
 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcT
anh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*d^(7/2)*f^(7/2))
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{\int \frac{(a+bx)\left(-\frac{1}{2}b(4bcCe+aCde+acCf-6Abdf)+\frac{1}{2}b(6bBdf-2aCdf-5bC(}{\sqrt{c+dx}\sqrt{e+fx}}}{3b^2df}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+\right.}{}$$

Mathematica [A] time = 2.01816, size = 379, normalized size = 1.02

$$\frac{\sqrt{e+fx}\left(3\sqrt{de-cf}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(b\left(2df\left(4Adf(cf+de)-B\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+C\left(3c^2def^2+5c^3f^3+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[e + f*x]*(-((d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(6*a*d*f*(4*B*d*f + C*
(-3*d*e - 3*c*f + 2*d*f*x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f
*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f
^2*x^2)))))/Sqrt[(d*(e + f*x))/(d*e - c*f)]) + 3*Sqrt[d*e - c*f]*(-2*a*d*f*
(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) +
 b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*(4*A*
d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcSinh[(Sqrt[f]
*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(24*d^3*f^(7/2)*(-(d*e) + c*f)*Sqrt[(d*(
e + f*x))/(d*e - c*f)])
```

Maple [B] time = 0.03, size = 1199, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/48*(18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*b*d^3*e^2*f+18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d*f^3+18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e^2*f+48*A*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*f^2+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*a*d^2*f^2-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e*f^2-24*B*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^2*f^3-24*B*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d
^3*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*b*c^2*d*f^3+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*f
^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e^2+28*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*b*c*d*e*f-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*
c*d*f^2-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^2*e*f+48*A*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*f^
3-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d*f^2-36*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b*d^2*e*f-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d
*f^2-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^2*e*f+12*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e*f^2
+12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*a*c*d^2*e*f^2-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*e*f^2-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e^2*f+24*B*(d*f)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*x*b*d^2*f^2+24*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*x*a*d^2*f^2-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*b*c^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^3+16*C*x^2*b*d^2*f^2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^3/d^3/(d*f)^(1/2)
/((d*x+c)*(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.98089, size = 1631, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/96*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2*f + (3*C*b
*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b*c^3 - 16*A
*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*b*d^3*f^
3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5
*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*
f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e)
)/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2
*f + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b
*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(-d
*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)
/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8*C*b*d^3*f^3*x^2 +
15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5*C*b*c^2
*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*f^2 + (5
*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f
^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [A] time = 3.43927, size = 603, normalized size = 1.63

$$\left(\sqrt{(dx+c)df - cdf + d^2e}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)Cb}{d^4f} - \frac{13\,Cbcd^{11}f^4 - 6\,Cad^{12}f^4 - 6\,Bbd^{12}f^4 + 5\,Cbd^{12}f^3e}{d^{15}f^5}\right) + \frac{3\left(11\,Cbc^2d^{11}f^4 - 10\,Cacd\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C*b/(d^4*f) - (13*C*b*c*d^11*f^4 - 6*C*a*d^12*f^4 - 6*B*b*d^12*f^4 +
 5*C*b*d^12*f^3*e)/(d^15*f^5)) + 3*(11*C*b*c^2*d^11*f^4 - 10*C*a*c*d^12*f^4
 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4 + 8*C*b*c*d^12*f^3*e
 - 6*C*a*d^13*f^3*e - 6*B*b*d^13*f^3*e + 5*C*b*d^13*f^2*e^2)/(d^15*f^5)) +
3*(5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*f^3 + 8*B*a*c*d^2*f^3 + 8*
A*b*c*d^2*f^3 - 16*A*a*d^3*f^3 + 3*C*b*c^2*d*f^2*e - 4*C*a*c*d^2*f^2*e - 4*
B*b*c*d^2*f^2*e + 8*B*a*d^3*f^2*e + 8*A*b*d^3*f^2*e + 3*C*b*c*d^2*f*e^2 - 6
*C*a*d^3*f*e^2 - 6*B*b*d^3*f*e^2 + 5*C*b*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d
*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^3))*d/abs(
d)
```

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cd}{4d^2f^2}$$

```
[Out] -((3*C*d*e + 5*c*C*f - 4*B*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*d^2*f^2) +
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) + ((C*(3*d^2*e^2 + 2*c*d*e*f +
3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2))
```

Rubi [A] time = 0.149335, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cd}{4d^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] -((3*C*d*e + 5*c*C*f - 4*B*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*d^2*f^2) +
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) + ((C*(3*d^2*e^2 + 2*c*d*e*f +
3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2))
```


Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\int \frac{\frac{1}{2}\left(-3cCde-c^2Cf+4Ad^2f\right)-\frac{1}{2}d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2d^2f}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2cdef+3c^2f^2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2cdef+3c^2f^2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2cdef+3c^2f^2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2cdef+3c^2f^2\right.\right.}{}$$

Mathematica [A] time = 0.765654, size = 173, normalized size = 1.05

$$\frac{\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+d\sqrt{f}\sqrt{c+dx}(e+fx)(4Ba}{4d^3f^{5/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x))
 + Sqrt[d*e - c*f]*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
- B*(d*e + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c +
 d*x])/Sqrt[d*e - c*f]])/(4*d^3*f^(5/2)*Sqrt[e + f*x])
```

Maple [B] time = 0.02, size = 425, normalized size = 2.6

$$\frac{1}{8d^2f^2}\left(8A\ln\left(1/2\,\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\sqrt{df}+cf+de}{\sqrt{df}}\right)d^2f^2-4B\ln\left(1/2\,\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\sqrt{df}+c}{\sqrt{df}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/8*(8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e*f+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d*e*f+3*C*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
d^2*e^2+4*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d*f+8*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*d*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*f-6*C*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2
)/f^2/d^2/((d*x+c)*(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51424, size = 879, normalized size = 5.36

$$\left[\frac{\left(3\,Cd^2e^2 + 2\left(Ccd - 2\,Bd^2\right)ef + \left(3\,Cc^2 - 4\,Bcd + 8\,Ad^2\right)f^2\right)\sqrt{df}\log\left(8\,d^2f^2x^2 + d^2e^2 + 6\,cdef + c^2f^2 + 4\left(2\,dfx + d\right.\right.}{16\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^
2)*f^2)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*
d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d
*f^2)*x) + 4*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^2)*f^2)*sqrt(d
*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*
e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d
*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (
d^2*e*f + c*d*f^2)*x)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^
2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [A] time = 3.6856, size = 262, normalized size = 1.6

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\sqrt{dx+c}\left(\frac{2(dx+c)C}{d^3f}-\frac{5Ccd^5f^2-4Bd^6f^2+3Cd^6fe}{d^8f^3}\right)-\frac{\left(3Cc^2f^2-4Bcdf^2+8Ad^2f^2+2Ccdfe-4Bd^2fe+3Cd^2e^2\right)\log\left(\left|-\sqrt{\cdots}\right|\right)}{\sqrt{df}d^2f^2}\right)}{4|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3*
f) - (5*C*c*d^5*f^2 - 4*B*d^6*f^2 + 3*C*d^6*f*e)/(d^8*f^3)) - (3*C*c^2*f^2
- 4*B*c*d*f^2 + 8*A*d^2*f^2 + 2*C*c*d*f*e - 4*B*d^2*f*e + 3*C*d^2*e^2)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(
d*f)*d^2*f^2))*d/abs(d)
```

3.57 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}} \, dx$

Optimal. Leaf size=188

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}}+\frac{C\sqrt{c+dx}\sqrt{e}}{bdf}$$

```
[Out] (C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f -
2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^2*d^
(3/2)*f^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[b*c - a*d]*Sqrt[b*e -
a*f])
```

Rubi [A] time = 0.340917, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 157, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}}+\frac{C\sqrt{c+dx}\sqrt{e}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] (C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f -
2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^2*d^
(3/2)*f^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[b*c - a*d]*Sqrt[b*e -
a*f])
```


Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \frac{\int \frac{\frac{1}{2}b(2Abdf-aC(de+cf))-\frac{1}{2}b(2aCdf+b(Cde+cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^2df}$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \left(A-\frac{a(bB-aC)}{b^2}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx + \frac{(-2aCdf-b \cdots}{\cdots}$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \left(2\left(A-\frac{a(bB-aC)}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{-bc+ad-(-be+af)x^2}\,dx, x, \cdots\right.$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{\sqrt{bc-ad}\sqrt{be-af}} + \frac{(-2aCdf-b(Cde+cC \cdots}{\cdots}$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}} - \frac{2\left(A-\frac{a(bB \cdots}{b \cdots}\right.}{\cdots}$$

Mathematica [A] time = 0.999701, size = 304, normalized size = 1.62

$$\frac{2\left(\frac{\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\sqrt{be-af}} - \frac{\sqrt{e+fx}(aCf-bBf+bCe)\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{f^{3/2}\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}} + \frac{bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de-cf}}+\sqrt{de-cf}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\right)}{2df^{3/2}\sqrt{\frac{d(e+fx)}{de-cf}}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(-(((b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x]
)/Sqrt[d*e - c*f]])/(f^(3/2)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)
])) + (b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c
*f)] + Sqrt[d*e - c*f]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(
2*d*f^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + ((A*b^2 + a*(-(b*B) + a*C))*
ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])
/(Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f])))/b^2
```

Maple [B] time = 0.031, size = 746, normalized size = 4.

$$-\frac{1}{2\,dfb^3}\left(2\,A\ln\left(\frac{1}{bx+a}\left(-2\,adfx+bcfx+bdex+2\,\sqrt{\frac{a^2df-abcf-abde+ceb^2}{b^2}}\sqrt{(dx+c)\left(fx+e\right)}b-acf-ade+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^2*d
*f*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+
a))*a*b*d*f*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)+2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2
*d*f*(d*f)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a*b*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
+C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*b^2*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d*e*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*b^2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2))*(f*x+e)^(1/2)*(d*x+
c)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/d/(d*f)^(1/2)/f/b^3/((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx + Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [B] time = 1.6315, size = 467, normalized size = 2.48

$$\frac{\sqrt{(dx+c)df - cdf + d^2e}\sqrt{dx+c}C|d|}{bd^3f} - \frac{2\left(\sqrt{df}Ca^2d^2 - \sqrt{df}Babd^2 + \sqrt{df}Ab^2d^2\right)\arctan\left(-\frac{bcdf - 2ad^2f + bd^2e - \left(\sqrt{df}\sqrt{dx+c} - \sqrt{(d}\right.}{2\sqrt{abcdf^2 - a^2d^2f^2 - b^2cdfe}}\right.}{\sqrt{abcdf^2 - a^2d^2f^2 - b^2cdfe + abd^2fe}b^2d|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*C*abs(d)/(b*d^3*f) - 2*(s
qrt(d*f)*C*a^2*d^2 - sqrt(d*f)*B*a*b*d^2 + sqrt(d*f)*A*b^2*d^2)*arctan(-1/2
*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e +
 a*b*d^2*f*e)*d))/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f
*e)*b^2*d*abs(d)) + 1/2*(sqrt(d*f)*C*b*c*f + 2*sqrt(d*f)*C*a*d*f - 2*sqrt(d
*f)*B*b*d*f + sqrt(d*f)*C*b*d*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^2)/(b^2*d*f^2*abs(d))
```

3.58 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=254

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-3a^2bC(cf+de)+2a^3Cdf+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b^2(bc-ad)^{3/2}(be-af)^{3/2}} -$$

```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) + (2*C*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(b^2*Sqrt[d]*Sqrt[f]) + ((2*a^3*C*d*f - 3*a^2*b*C*(d*e + c*f) - b
^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e + B*d*e + B*c*f - 2*A*d*f))*A
rcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b
^2*(b*c - a*d)^(3/2)*(b*e - a*f)^(3/2))
```

Rubi [A] time = 0.638489, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1613, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-3a^2bC(cf+de)+2a^3Cdf+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b^2(bc-ad)^{3/2}(be-af)^{3/2}} -$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) + (2*C*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(b^2*Sqrt[d]*Sqrt[f]) + ((2*a^3*C*d*f - 3*a^2*b*C*(d*e + c*f) - b
^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e + B*d*e + B*c*f - 2*A*d*f))*A
rcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b
^2*(b*c - a*d)^(3/2)*(b*e - a*f)^(3/2))
```


Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{-\frac{a^2C(de+cf)+b^2(2Bce-Ade-Acf)-ab(2cCe+Bde+Bcf-2Adf)}{2b} - \frac{C\cdots}{\cdots}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\cdots}{(bc-ad)(be-af)}\cdots$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{C\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^2} - \frac{\left(2a^3Cdf-3a^2bC(de+\cdots\right.}{\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2d} - \cdots$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{\left(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade\cdots\right.}{b^2\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}} + \frac{\left(2a^3Cdf-3a^2bC(\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.01084, size = 324, normalized size = 1.28

$$\frac{-\frac{b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)(bc-ad)(be-af)} + \frac{\left(a(aC-bB)+Ab^2\right)(-2adf+bcf+bde)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(be-af)^{3/2}} + \frac{2(bB-2aC)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\sqrt{be-af}} + \frac{2C\sqrt{e+fx}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{\cdots}}{\sqrt{de\cdots}}\right)}{\sqrt{f}\sqrt{de-cf}\sqrt{\frac{d(e+fx\cdots}{de-cf\cdots}}}\cdots}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*
(b*e - a*f)*(a + b*x))) + (2*C*Sqrt[e + f*x]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x]
)/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)
]) + (2*(b*B - 2*a*C)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) +
 a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f]) + ((A*b^2 + a*(
-(b*B) + a*C))*(b*d*e + b*c*f - 2*a*d*f)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d
*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/((-(b*c) + a*d)^(3/2)*(b*e - a*f)
^(3/2)))/b^2
```

Maple [B] time = 0.052, size = 2973, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-2*B*a*b^3*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-B*ln((-2*a*d*f*x+b*c*f*
x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a*b^3*c*f*(d*f)^(1/2)-B*ln((-2*a*d
*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a*b^3*d*e*(d*f)^(1/2)-2
*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*x*a^2*b^2*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*a*b^
3*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*x*a*b^3*d*e*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+
2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a
*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a^3*b*d*f*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a^2*b^2*c*f*(d*f)^(1/2)+3*C*ln(
(-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a^2*b^2*d*e*(d*f
)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*a
*b^3*c*e*(d*f)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*x*a*b^3*d*f*(d*f)^(1/2)+2*A*b^4*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*c*f*(d*f)^(1/2)-A*ln((-2*a*d*f*x+
b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*d*e*(d*f)^(1/2)+2*B*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*x*b^4*c*e*(d*f)^(1/
2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*x*b^4*c*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*A*ln((-2*
a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*f*(d*f)^(1/2
)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*(
```

```
d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*
b^3*d*e*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*a^2*b^2*c*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+
2*b*c*e)/(b*x+a))*a^2*b^2*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*
x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d*f*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*c*f*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a*b^3*c*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f*
(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))
*a^3*b*d*e*(d*f)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*
e)/(b*x+a))*a^2*b^2*c*e*(d*f)^(1/2)+2*C*a^2*b^2*(d*f)^(1/2)*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-2*C*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d*f*(d*f)^(1/2))/((d*x+c)*(
f*x+e))^(1/2)/(a*d-b*c)/(d*f)^(1/2)/(a*f-b*e)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)/(b*x+a)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 11.1543, size = 1831, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 2*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*A*a*b^2*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 3*sqrt(d*f)*C*a^2*b*d^3*e -
sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2*a
*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d
))/((a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs(d)*e + a*b^3*d*abs(d
)*e)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 2*(sq
rt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b
^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*
f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*c*d^2*f - s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*
A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^2*d^3*f + sqrt
(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2
 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*
d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*
f - c*d*f + d^2*e))^4*b)*(a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs
(d)*e + a*b^3*d*abs(d)*e)) - sqrt(d*f)*C*log((sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2)/(b^2*f*abs(d))
```

3.59 $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=424

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+\right.\right.}{4(bc-ad)^{5/2}(be-af)^{5}}$$

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^2) + ((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f - 6*
A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e + c*f)
))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))
 - ((b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*
A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^2 -
 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(
2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*
c - a*d]*Sqrt[e + f*x])])/(4*(b*c - a*d)^(5/2)*(b*e - a*f)^(5/2))
```

Rubi [A] time = 0.967372, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1613, 151, 12, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+\right.\right.}{4(bc-ad)^{5/2}(be-af)^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^2) + ((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f - 6*
A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e + c*f)
))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))
 - ((b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*
A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^2 -
 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(
2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*
c - a*d]*Sqrt[e + f*x])])/(4*(b*c - a*d)^(5/2)*(b*e - a*f)^(5/2))
```


Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(4cCe+Bde+Bcf-4Adf)+b^2(4Bce-3A(de+cf}{2b}}{(a+bx)^2\sqrt{c}}}{2(bc-ad}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Ad\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Ad\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Ad\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Ad\right.}{}$$

Mathematica [A] time = 2.73413, size = 513, normalized size = 1.21

$$\frac{\frac{\left(a(aC-bB)+Ab^2\right)\left(\frac{3b\sqrt{c+dx}\sqrt{e+fx}(-2adf+bcf+bde)}{(a+bx)(bc-ad)(be-af)}-\frac{\left(8a^2d^2f^2-8abdf(cf+de)+b^2\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(be-af)^{3/2}}\right)}{(bc-ad)(be-af)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)^2(bc-ad)(be-af)}-}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)
*(b*e - a*f)*(a + b*x)^2) - (4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])
/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (8*C*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c
+ d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*Sqrt[b*e -
 a*f]) + (4*(b*B - 2*a*C)*(b*d*e + b*c*f - 2*a*d*f)*ArcTan[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/((-(b*c) + a*d)^(3/2)*
(b*e - a*f)^(3/2)) + ((A*b^2 + a*(-(b*B) + a*C))*((3*b*(b*d*e + b*c*f - 2*a
*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) - ((
8*a^2*d^2*f^2 - 8*a*b*d*f*(d*e + c*f) + b^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*
f^2))*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f
*x])])/((-(b*c) + a*d)^(3/2)*(b*e - a*f)^(3/2))))/((b*c - a*d)*(b*e - a*f))
)/(4*b^2)
```

Maple [B] time = 0.107, size = 7119, normalized size = 16.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=826

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(bc-ad)(be-af)(a+bx)^3}+\frac{\left(-2df\left(C\left(3d^2e^2+2cdfe+3c^2f^2\right)+4df(2Adf-B(de+cf))\right)a^3+b\left(C\left(d^3e^3\right.\right.\cdots}{\cdots}$$

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^3) + ((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*f - 1
0*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*e + c*
f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(12*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*
x)^2) + ((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(15
*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2))
 - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 -
 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^2) +
2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b*(b*
c - a*d)^3*(b*e - a*f)^3*(a + b*x)) + ((b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2*B
*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*
f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) - c*d
^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f + 18*A*
f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f -
 B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 + c^
3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))))
*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/
(8*(b*c - a*d)^(7/2)*(b*e - a*f)^(7/2))
```

Rubi [A] time = 2.43334, antiderivative size = 826, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1613, 151, 12, 93, 208}

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(bc-ad)(be-af)(a+bx)^3}+\frac{\left(-2df\left(C\left(3d^2e^2+2cdfe+3c^2f^2\right)+4df(2Adf-B(de+cf))\right)a^3+b\left(C\left(d^3e^3\right.\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^3) + ((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*f - 1
0*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*e + c*
f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(12*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*
x)^2) + ((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(15
*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2))
 - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 -
 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^2) +
2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b*(b*
c - a*d)^3*(b*e - a*f)^3*(a + b*x)) + ((b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2*B
*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*
f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) - c*d
^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f + 18*A*
f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f -
 B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 + c^
3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))))
*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/
```

```
(8*(b*c - a*d)^(7/2)*(b*e - a*f)^(7/2))
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(6cCe+Bde+Bcf-6Adf)+b^2(6Bce-5A(de+cf))}{2b}+\left(\right.}{(a+bx)^3\sqrt{c+d}}}{3(bc-ad)(}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf\right.}{12}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf\right.}{12}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf\right.}{12}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf\right.}{12}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf\right.}{12}$$

Mathematica [A] time = 6.08097, size = 800, normalized size = 0.97

$$-\frac{-24C(bc-ad)^2(be-af)^2(bde+bcf-2adf)\tan^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{ad-bc}\sqrt{e+fx}}\right)(a+bx)^3-6(bB-2aC)(be-af)\left(3b(ad-bc)^{3/2}\sqrt{be\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(-8*b*(A*b^2 + a*(-(b*B) + a*C))*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^(5/2)*Sq
rt[c + d*x]*Sqrt[e + f*x] - 12*b*(b*B - 2*a*C)*(-(b*c) + a*d)^(5/2)*(b*e -
a*f)^(5/2)*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x] - 24*b*C*(-(b*c) + a*d)^(5
/2)*(b*e - a*f)^(5/2)*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x] - 24*C*(b*c -
 a*d)^2*(b*e - a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^3*ArcTan[(Sqrt[b*
e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])] - 6*(b*B - 2*a*
C)*(b*e - a*f)*(a + b*x)^2*(3*b*(-(b*c) + a*d)^(3/2)*Sqrt[b*e - a*f]*(b*d*e
 + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x] - (b*c - a*d)*(8*a^2*d^2*f^
2 - 8*a*b*d*f*(d*e + c*f) + b^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*(a + b
*x)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x
])]) - (A*b^2 + a*(-(b*B) + a*C))*(a + b*x)*(10*b*(-(b*c) + a*d)^(3/2)*(b*e
 - a*f)^(3/2)*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x] - (a +
b*x)*(-(b*Sqrt[-(b*c) + a*d]*Sqrt[b*e - a*f]*(44*a^2*d^2*f^2 - 44*a*b*d*f*(
d*e + c*f) + b^2*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x]*Sqrt
[e + f*x]) - 3*(b*d*e + b*c*f - 2*a*d*f)*(8*a^2*d^2*f^2 - 8*a*b*d*f*(d*e +
c*f) + b^2*(5*d^2*e^2 - 2*c*d*e*f + 5*c^2*f^2))*(a + b*x)*ArcTan[(Sqrt[b*e
- a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])))/(24*b^2*(-(b*c
) + a*d)^(7/2)*(b*e - a*f)^(7/2)*(a + b*x)^3)
```

Maple [B] time = 0.246, size = 18802, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.61 $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=1182

result too large to display

```
[Out] (2*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b^2*d
*f^2*((c^2*C - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3*c^2
*d*e*f^2 - 8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c*d*e
*f - 4*c^2*f^2))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(315*b^3*d^3*
f^3) - (2*(7*b*d*f*(b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*d*f) + (a*d*f - 4*b
*(d*e + c*f))*(2*a*C*d*f - b*(3*B*d*f - 2*C*(d*e + c*f))))*Sqrt[a + b*x]*Sq
rt[c + d*x]*(e + f*x)^(3/2))/(105*b^2*d^2*f^3) - (2*(2*a*C*d*f - b*(3*B*d*f
 - 2*C*(d*e + c*f)))*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(21*b*d
^2*f^2) + (2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) -
 (2*Sqrt[-(b*c) + a*d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f +
 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^
2*e^2 - c*d*e*f + c^2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c
^2*d*e*f^2 + 8*c^3*f^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*
d*e*f + 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*
f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2
*f^2) - B*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(
b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/
2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*
d]*(b*e - a*f)*(d*e - c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*
f - 4*B*d*f) - 3*a*b^2*d*f^2*((c^2*C - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^
3*(C*(16*d^3*e^3 - 3*c^2*d*e*f^2 - 8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f
) - B*(8*d^2*e^2 - c*d*e*f - 4*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sq
rt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*S
qrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 4.1659, antiderivative size = 1154, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}+\frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2}-\frac{2(7bdf(bcCe+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*((8*a^3*C*d*f)/b - 3*a*b*(B*d*e - 2*B*c*f + (c^2*C*f)/d - 7*A*d*f) + 3*a
^2*(C*d*e - c*C*f - 4*B*d*f) + b^2*((3*c^2*C*e)/d - 42*A*d*e - (16*C*d*e^3)
/f^2 + 21*A*c*f + (8*c^3*C*f)/d^2 - B*(3*c*e - (24*d*e^2)/f + (12*c^2*f)/d)
))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(315*b^2*d*f) - (2*(7*b*d*f*(
b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*d*f) - (a*d*f - 4*b*(d*e + c*f))*(3*b*B
*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x
)^(3/2))/(105*b^2*d^2*f^3) + (2*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))
*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(21*b*d^2*f^2) + (2*C*(a +
b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) - (2*Sqrt[-(b*c) + a*
d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^
```

```
2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^
2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + 8*c^3*f
^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*d*e*f + 5*c^2*f^2)))
 + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3
+ 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) - B*(8*d^3*e^3
 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c -
a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) +
a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e
- c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b
^2*d*f^2*((c^2*C - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3
*c^2*d*e*f^2 - 8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c
*d*e*f - 4*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(
b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], (
(b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e
+ f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{2\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(-\ldots\right.}{\ldots}$$

$$= \frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2} + \frac{2C\ldots}{\ldots}$$

$$= -\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf\ldots}{105b^2d^2f^3\ldots}$$

$$= \frac{2\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2\left(\left(c^2C-7Ad^2\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2\left(\left(c^2C-7Ad^2\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2\left(\left(c^2C-7Ad^2\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2\left(\left(c^2C-7Ad^2\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2\left(\left(c^2C-7Ad^2\ldots\right.\right.\right.}{\ldots}$$

Mathematica [C] time = 17.7168, size = 11933, normalized size = 10.1

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.088, size = 14778, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\left(A + Bx + Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cx^2 + Bx + A\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

3.62 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)+b^2\left(-\left(7df(-5Adf-\right.\right.\right.}{105b^4d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (-2*(5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f)) - (2*b*d*e - b*c*f +
 4*a*d*f)*(6*a*C*d*f - b*(7*B*d*f - 4*C*(d*e + c*f))))*Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x])/(105*b^3*d^2*f^2) - (2*(6*a*C*d*f - b*(7*B*d*f - 4*C
*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*d*f^2)
+ (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - (2*Sqrt[-
(b*c) + a*d]*(3*b*d*f*(5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f)) -
(b*c*e + a*d*e + 3*a*c*f)*(6*a*C*d*f - b*(7*B*d*f - 4*C*(d*e + c*f)))) + 2*
((b*d*e)/2 - (b*c + a*d)*f)*(5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*
f)) - (2*b*d*e - b*c*f + 4*a*d*f)*(6*a*C*d*f - b*(7*B*d*f - 4*C*(d*e + c*f)
))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(10
5*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[
-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e
 - 5*c*C*f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*
e^2 - c*d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f
*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*
d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*S
qrt[e + f*x])
```

Rubi [A] time = 2.23008, antiderivative size = 769, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)+b^2\left(-\left(7df(-5Adf-\right.\right.\right.}{105b^4d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*f - 4*b*C*(d*e + c*f
)))/(b*d*f) + 5*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f)))*Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d*f) + (2*(7*b*B*d*f - 6*a*C*d*f - 4*b*
C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*d*f^2)
+ (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - (2*Sqrt[-
(b*c) + a*d]*(3*b*d*f*((b*c*e + a*d*e + 3*a*c*f)*(7*b*B*d*f - 6*a*C*d*f - 4
*b*C*(d*e + c*f)) + 5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))) + 2*
((b*d*e)/2 - (b*c + a*d)*f)*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C
*d*f - 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f
))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(10
5*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[
-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e
 - 5*c*C*f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*
```

```
e^2 - c*d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f
*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*
d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*S
qrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}dx = \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} + \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{1}{2}b(3aC(de+cf)+b(cCe-7Adf)\right.}{\sqrt{a+bx}}}{7b^2df}$$

$$= \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2} + \frac{2C\sqrt{a+b}}{}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{105b^3d^2f^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{105b^3d^2f^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{105b^3d^2f^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{105b^3d^2f^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{105b^3d^2f^2}$$

Mathematica [C] time = 13.3288, size = 917, normalized size = 1.18

$$-\frac{2\left(\sqrt{\frac{bc}{d}-a}\left(\left(C\left(-8d^3e^3+5cd^2fe^2+5c^2df^2e-8c^3f^3\right)-7df\left(5Adf(de+cf)-2B\left(d^2e^2-cdfe+c^2f^2\right)\right)\right)b^3+adf\right(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f +
2*C*(d*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*
```

```
d^2*e^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*
c^2*d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*
e*f + c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x
)*(c + d*x)*(e + f*x)*(-24*a^2*C*d^2*f^2 + a*b*d*f*(28*B*d*f + C*(5*d*e + 5
*c*f + 18*d*f*x)) + b^2*(-7*d*f*(B*c*f + 5*A*d*f + B*d*(e + 3*f*x)) + C*(4*
c^2*f^2 - c*d*f*(2*e + 3*f*x) + d^2*(4*e^2 - 3*e*f*x - 15*f^2*x^2)))) + I*(
b*c - a*d)*f*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f + 2*C*(d*e + c*f)
) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*d^2*e^2 + 8*c*d
*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8
*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*e*f + c^2*f^2))
))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*
(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e -
 a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(24*a^2*C*d^2*f^2
+ a*b*d*f*(-5*C*d*e + 13*c*C*f - 28*B*d*f) + b^2*(7*d*f*(B*d*e - 2*B*c*f +
5*A*d*f) - C*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(1
05*b^5*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]
)
```

Maple [B] time = 0.042, size = 10271, normalized size = 13.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{\sqrt{bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{\sqrt{bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

3.63 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=706

$$-\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cdf^2-abf(20Bdf+cCf+7Cde)+b^2(5df(3Af+Be)-Ce(2de-cf))\right)\text{El}}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e +
 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(1
5*b^3*d*f*(b*e - a*f)) + (2*(6*a^2*C*d*f + b^2*(c*C*e + 5*A*d*f) - a*b*(C*d
*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b^2*
(b*c - a*d)*f*(b*e - a*f)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) +
a*d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(
B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/
Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*
Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e
 - c*f)*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(
B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c
) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.84465, antiderivative size = 706, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 154, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+\right.\right.\right.}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x]
```


```
[Out] (2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e +
 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(1
5*b^3*d*f*(b*e - a*f)) + (2*(6*a^2*C*d*f + b^2*(c*C*e + 5*A*d*f) - a*b*(C*d
*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b^2*
(b*c - a*d)*f*(b*e - a*f)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) +
a*d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(
B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/
Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*
Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e
 - c*f)*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(
B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c
) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
```

```
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(cCe+\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{2\left(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}(e+\cdots}{5b^2(bc-ad)f(be-af)}$$

$$= \frac{2\left(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf\cdots\right.}{15b^3df(be-af)}$$

$$= \frac{2\left(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf\cdots\right.}{15b^3df(be-af)}$$

$$= \frac{2\left(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf\cdots\right.}{15b^3df(be-af)}$$

$$= \frac{2\left(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf\cdots\right.}{15b^3df(be-af)}$$

$$= \frac{2\left(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf\cdots\right.}{15b^3df(be-af)}$$

Mathematica [C] time = 8.11424, size = 633, normalized size = 0.9

$$-\frac{2\left(-ibf(a+bx)^{3/2}(de-cf)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\left(24a^2Cd^2f-abd(20Bdf+7cCf+Cde)+b^2\left(15Ad^2f+cd(5Bf+Ce)-\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x
]
```

```
[Out] (-2*(-(b^2*Sqrt[-a + (b*c)/d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f
+ 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f
+ c^2*f^2)))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e
 + f*x)*(15*(A*b^2 + a*(-(b*B) + a*C))*d*f - (-9*a*C*d*f + b*(C*d*e + c*C*f
 + 5*B*d*f))*(a + b*x) - 3*b*C*d*f*x*(a + b*x)) - I*(b*c - a*d)*f*(48*a^2*C
*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f
```

```
+ 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*
b*f*(d*e - c*f)*(24*a^2*C*d^2*f - a*b*d*(C*d*e + 7*c*C*f + 20*B*d*f) + b^2*
(-2*c^2*C*f + 15*A*d^2*f + c*d*(C*e + 5*B*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
 d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh
[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*
b^5*Sqrt[-a + (b*c)/d]*d^2*f^2*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.051, size = 6257, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^2x^2 + 2\,abx + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^2*x
^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}\left(A + Bx + Cx^2\right)}{(a + bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(3/2), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

3.64 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=687

$$\frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right.\right.}{3b^4\sqrt{df}\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}}$$

```
[Out] (2*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*
B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^3*(b*c - a*d)*(b*e
- a*f)) - (2*(b*B - 2*a*C)*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b^2*(b*e - a*f)*
Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2)
)/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/2)) + (2*(16*a^3*C*d^2*f^2 - 8*
a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e
^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*
e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]
*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[
c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(8*a^2*C*d*f + b
^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.90276, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1614, 150, 154, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(-8a^2bdf(Bdf+2C(cf+de))+16a^3Cd^2f^2+ab^2\left(df(2Adf+7Bcf+7Bde)+C\left(c^2f^2+16cdef\right.\right.\right.}{3b^4\sqrt{df}\sqrt{c+dx}\sqrt{ad-bc}(be-af)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*
B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^3*(b*c - a*d)*(b*e
- a*f)) - (2*(b*B - 2*a*C)*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b^2*(b*e - a*f)*
Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2)
)/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/2)) + (2*(16*a^3*C*d^2*f^2 - 8*
a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e
^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*
e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]
*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[
c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(8*a^2*C*d*f + b
^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}}dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - 2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3\left(b^2Bce+a^2C(de+\right.}{}\right.}{}$$

$$= -\frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{}}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{}}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{}}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{}}{3b^3(bc-ad)(be-af)}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{}}{3b^3(bc-ad)(be-af)}$$

Mathematica [C] time = 13.4094, size = 938, normalized size = 1.37

$$\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2C}{3b^3} - \frac{2\left(-8Cdfa^3+7bCdea^2+7bcCfa^2+5bBdfa^2-6b^2cCea-4b^2Bdea-4b^2Bcfa-2A\right.}{3b^3(bc-ad)(be-af)(a+bx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x
]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((2*C)/(3*b^3) - (2*(A*b^2 - a*b*
B + a^2*C))/(3*b^3*(a + b*x)^2) - (2*(3*b^3*B*c*e - 6*a*b^2*c*C*e + A*b^3*d
*e - 4*a*b^2*B*d*e + 7*a^2*b*C*d*e + A*b^3*c*f - 4*a*b^2*B*c*f + 7*a^2*b*c*
C*f - 2*a*A*b^2*d*f + 5*a^2*b*B*d*f - 8*a^3*C*d*f))/(3*b^3*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) - (2*(a + b*x)^(3/2)*(-(Sqrt[-a + (b*c)/d]*(-16*a^3*C*d
^2*f^2 + 8*a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^3*(c^2*C*e*f + A*d^2*e*f
 + c*d*(C*e^2 + 6*B*e*f + A*f^2)) - a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f
) + C*(d^2*e^2 + 16*c*d*e*f + c^2*f^2)))*(d + (b*c)/(a + b*x) - (a*d)/(a +
b*x))*(f + (b*e)/(a + b*x) - (a*f)/(a + b*x))) + (I*(-(b*c) + a*d)*f*(-16*a
^3*C*d^2*f^2 + 8*a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^3*(c^2*C*e*f + A*d
^2*e*f + c*d*(C*e^2 + 6*B*e*f + A*f^2)) - a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2
*A*d*f) + C*(d^2*e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[1 - a/(a + b*x) + (b*c)
/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*EllipticE[I*Arc
Sinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/S
qrt[a + b*x] + (I*b*(-(b*c) + a*d)*f*(d*e - c*f)*(8*a^2*C*d*f + b^2*(c*C*e
+ 3*B*d*e + A*d*f) - a*b*(7*C*d*e + c*C*f + 4*B*d*f))*Sqrt[1 - a/(a + b*x)
+ (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*Elliptic
F[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d
*f)])/Sqrt[a + b*x]))/(3*b^5*Sqrt[-a + (b*c)/d]*d*(b*c - a*d)*f*(b*e - a*f)
*Sqrt[c + ((a + b*x)*(d - (a*d)/(a + b*x)))/b]*Sqrt[e + ((a + b*x)*(f - (a*
f)/(a + b*x)))/b])
```

Maple [B] time = 0.096, size = 16177, normalized size = 23.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{\left(bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^3*x
^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

3.65 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=964

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}+\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf\right.}{15b^2(bc-ad)(be-af)^2(a+b}$$

```
[Out] (2*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*
C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f
) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^3*(b*c - a*
d)^2*(b*e - a*f)*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*B*d
*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*f +
8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(15*b^2*(b*c - a*d)*(b*e -
 a*f)^2*(a + b*x)^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e +
f*x)^(3/2))/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(48*
a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*
f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d*f*(2*A*d*f
- 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipti
cE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(
b*e - a*f))])/(15*b^4*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt
[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*b*d*(23
*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A
*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqr
t[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcS
in[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a
*f))])/(15*b^4*Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[
e + f*x])
```

Rubi [A] time = 3.11586, antiderivative size = 964, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 150, 158, 114, 113, 121, 120}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}+\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf\right.}{15b^2(bc-ad)(be-af)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x]
```

```
[Out] (2*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*
C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f
) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^3*(b*c - a*
d)^2*(b*e - a*f)*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*B*d
*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*f +
8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(15*b^2*(b*c - a*d)*(b*e -
 a*f)^2*(a + b*x)^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e +
f*x)^(3/2))/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(48*
a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e
```

```
*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*
f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d*f*(2*A*d*f
- 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipti
cE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(
b*e - a*f))])/(15*b^4*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt
[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*b*d*(23
*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A
*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqr
t[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcS
in[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a
*f))])/(15*b^4*Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[
e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
```

```
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(5cCe\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{2\left(6a^3Cdf+ab^2(10cCe+3Bde+3Bcf-4Adf)-b^3(5Bce-2A(de+cf))-a\cdots\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}\cdots}$$

$$= \frac{2\left(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3\left(15c^2Ce-2Ad^2e+cd(5Be\cdots\right.\right.}{15b^3(bc-ad)^2(b\cdots}$$

$$= \frac{2\left(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3\left(15c^2Ce-2Ad^2e+cd(5Be\cdots\right.\right.}{15b^3(bc-ad)^2(b\cdots}$$

$$= \frac{2\left(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3\left(15c^2Ce-2Ad^2e+cd(5Be\cdots\right.\right.}{15b^3(bc-ad)^2(b\cdots}$$

$$= \frac{2\left(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3\left(15c^2Ce-2Ad^2e+cd(5Be\cdots\right.\right.}{15b^3(bc-ad)^2(b\cdots}$$

$$= \frac{2\left(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3\left(15c^2Ce-2Ad^2e+cd(5Be\cdots\right.\right.}{15b^3(bc-ad)^2(b\cdots}$$

Mathematica [C] time = 16.847, size = 9529, normalized size = 9.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x
]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.209, size = 34395, normalized size = 35.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^4*x
^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

3.66 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=1716

result too large to display

```
[Out] (-2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4*B*d*f) - 3*a*b^3
*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C*e^2 - 5*B*e*f + 5
*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*(35*C*e^2 - 14*B*e*
f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d*f) - C*(15*d^2*e^2
+ 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*
d)^2*(b*e - a*f)^2*(a + b*x)^(3/2)) + (2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^
2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f
) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f +
8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f) - c^2
*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e + 20*A*
f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A*
d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c^
2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*e
*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*d)^3*(b*e
 - a*f)^3*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*B*d*e + 3*
B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*f - 10*C*(d
*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*(b*c - a*d)*(b*e - a*f)^
2*(a + b*x)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(
3/2))/(7*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(7/2)) + (2*Sqrt[d]*(48*a^5*C*
d^3*f^3 + 8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c
*d^2*e^2*(14*B*e + 5*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f
*(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3
*f^2*(7*C*e - B*f) - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C
*e^2 - f*(19*B*e + 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f +
103*c^2*f^2) + d*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3
+ 94*c*d^2*e^2*f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) -
 B*(3*d^2*e^2 + 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]],
 ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*f
)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)
*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(43*C*d*e + 61*c*C*f - 4*B*d*f) + b^4*(8*A*d
^2*e^2 - c*d*e*(14*B*e + A*f) + c^2*(35*C*e^2 + 7*B*e*f - 4*A*f^2)) + 3*a*b
^3*(d^2*e*(2*B*e - 5*A*f) - c^2*f*(28*C*e + B*f) - c*d*(14*C*e^2 - 5*B*e*f
- 3*A*f^2)) - 3*a^2*b^2*(d*f*(2*B*d*e + 3*B*c*f - A*d*f) - C*(5*d^2*e^2 + 3
7*c*d*e*f + 15*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*
f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 7.04535, antiderivative size = 1716, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1614, 150, 152, 158, 114, 113, 121, 120}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4*B*d*f) - 3*a*b^3
*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C*e^2 - 5*B*e*f + 5
*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*(35*C*e^2 - 14*B*e*
f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d*f) - C*(15*d^2*e^2
+ 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*
d)^2*(b*e - a*f)^2*(a + b*x)^(3/2)) + (2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^
2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f
) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f +
8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f) - c^2
*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e + 20*A*
f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A*
d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c^
2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*e
*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*d)^3*(b*e
 - a*f)^3*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*B*d*e + 3*
B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*f - 10*C*(d
*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*(b*c - a*d)*(b*e - a*f)^
2*(a + b*x)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(
3/2))/(7*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(7/2)) + (2*Sqrt[d]*(48*a^5*C*
d^3*f^3 + 8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c
*d^2*e^2*(14*B*e + 5*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f
*(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3
*f^2*(7*C*e - B*f) - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C
*e^2 - f*(19*B*e + 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f +
103*c^2*f^2) + d*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3
+ 94*c*d^2*e^2*f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) -
 B*(3*d^2*e^2 + 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]],
 ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*f
)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)
*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(43*C*d*e + 61*c*C*f - 4*B*d*f) + b^4*(8*A*d
^2*e^2 - c*d*e*(14*B*e + A*f) + c^2*(35*C*e^2 + 7*B*e*f - 4*A*f^2)) + 3*a*b
^3*(d^2*e*(2*B*e - 5*A*f) - c^2*f*(28*C*e + B*f) - c*d*(14*C*e^2 - 5*B*e*f
- 3*A*f^2)) - 3*a^2*b^2*(d*f*(2*B*d*e + 3*B*c*f - A*d*f) - C*(5*d^2*e^2 + 3
7*c*d*e*f + 15*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*
f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(7cCe\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{2\left(6a^3Cdf+ab^2(14cCe+3Bde+3Bcf-8Adf)-b^3(7Bce-4A(de+cf))+a\cdots\right.}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(24a^4Cd^2f^2-a^3bdf(61Cde+43cCf-4Bdf)-3ab^3\left(d^2e(Be-3Af)+2c^2\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 19.4264, size = 15719, normalized size = 9.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x
]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.36, size = 68345, normalized size = 39.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^5x^5 + 5\,ab^4x^4 + 10\,a^2b^3x^3 + 10\,a^3b^2x^2 + 5\,a^4bx + a^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^5*x
^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{dx + c}\sqrt{fx + e}}{(bx + a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

3.67 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1235

result too large to display

```
[Out] (-2*(5*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*
c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*(
(a*d*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*
A*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(945*b^2*d^3*f^4)
- (2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (6*b*d*e + 4*
b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f)))*Sqrt[a + b*
x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(315*b*d^3*f^3) - (2*(4*a*C*d*f + b*(8*C*
d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(6
3*b*d^2*f^2) + (2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f
) + (2*Sqrt[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*
C*f - 18*B*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) -
C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(2*C*(92*d^3*e^3 - 33*c*
d^2*e^2*f - 18*c^2*d*e*f^2 - 16*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f)
- B*(72*d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d
^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d
*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9
*c^2*d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))])/(315*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/
(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d^3*
f^3 + 3*a^2*b*d^2*f^2*(3*C*d*e - c*C*f - 3*B*d*f) - 3*a*b^2*d*f*(3*d*f*(16*
B*d*e + 3*B*c*f - 21*A*d*f) - 5*C*(8*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - b^3*
(C*(128*d^3*e^3 + 24*c*d^2*e^2*f + 15*c^2*d*e*f^2 + 8*c^3*f^3) + 3*d*f*(7*A
*d*f*(8*d*e + c*f) - 4*B*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3
15*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 4.39515, antiderivative size = 1235, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$\frac{2C(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^{5/2}}{9bdf}-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^{3/2}}{63bd^2f^2}-\frac{2(7bdf(5bcCe+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-2*(5*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*
c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*(
(a*d*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*
A*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(945*b^2*d^3*f^4)
- (2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (6*b*d*e + 4*
b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f)))*Sqrt[a + b*
```

```
x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(315*b*d^3*f^3) - (2*(4*a*C*d*f + b*(8*C*
d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(6
3*b*d^2*f^2) + (2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f
) + (2*Sqrt[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*
C*f - 18*B*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) -
C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(2*C*(92*d^3*e^3 - 33*c*
d^2*e^2*f - 18*c^2*d*e*f^2 - 16*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f)
- B*(72*d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d
^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d
*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9
*c^2*d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))])/(315*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/
(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d^3*
f^3 + 3*a^2*b*d^2*f^2*(3*C*d*e - c*C*f - 3*B*d*f) - 3*a*b^2*d*f*(3*d*f*(16*
B*d*e + 3*B*c*f - 21*A*d*f) - 5*C*(8*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - b^3*
(C*(128*d^3*e^3 + 24*c*d^2*e^2*f + 15*c^2*d*e*f^2 + 8*c^3*f^3) + 3*d*f*(7*A
*d*f*(8*d*e + c*f) - 4*B*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3
15*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
```

```
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{2\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\left(-\frac{1}{2}b(5bcCe+3aCde+acCf-\cdots\right.}{\sqrt{e\cdots}}}{9b^2\cdots}$$

$$= -\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2} + \frac{2C\cdots}{\cdots}$$

$$= -\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aC\cdots}{315bd^3f^3}$$

$$= -\frac{2\left(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(\cdots\right.}{\cdots}$$

$$= -\frac{2\left(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(\cdots\right.}{\cdots}$$

$$= -\frac{2\left(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(\cdots\right.}{\cdots}$$

$$= -\frac{2\left(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(\cdots\right.}{\cdots}$$

$$= -\frac{2\left(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(\cdots\right.}{\cdots}$$

Mathematica [C] time = 18.4476, size = 12483, normalized size = 10.11

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x
]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.056, size = 15857, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(bx + a)^{\frac{3}{2}}\sqrt{dx + c}}{\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cbx^3 + (Ca + Bb)x^2 + Aa + (Ba + Ab)x)\sqrt{bx + a}\sqrt{dx + c}}{\sqrt{fx + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b*x^3 + (C*a + B*b)*x^2 + A*a + (B*a + A*b)*x)*sqrt(b*x + a)*sq
rt(d*x + c)/sqrt(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(bx + a)^{\frac{3}{2}}\sqrt{dx + c}}{\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

3.68 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=766

$$\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cd^2f^2+abdf(-7Bdf-2cCf+8Cde)+b^2\left(-\left(7df(-10Adf+Bc\right.\right.\right.}{105b^3d^{5/2}f^4\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (-2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(
2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f)))*Sqrt[a + b*x]*
Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d^2*f^3) - (2*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(35*b*d
^2*f^2) + (2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - (
2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7
*A*b*d*f) - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7
*B*d*f))) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(
d*(b*e - a*f))])/(105*b^3*d^(5/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e
 - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2
+ a*b*d*f*(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*
A*d*f) - 4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^3*d^(5/2
)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 2.06141, antiderivative size = 766, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cd^2f^2+abdf(-7Bdf-2cCf+8Cde)+b^2\left(-\left(7df(-10Adf+Bc\right.\right.\right.}{105b^3d^{5/2}f^4\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(
2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f)))*Sqrt[a + b*x]*
Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d^2*f^3) - (2*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(35*b*d
^2*f^2) + (2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - (
2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7
*A*b*d*f) - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7
*B*d*f))) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(
d*(b*e - a*f))])/(105*b^3*d^(5/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e
 - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2
+ a*b*d*f*(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*
```

```
A*d*f) - 4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(105*b^3*d^(5/2
)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{1}{2}b(3bcCe+3aCde+acCf-7Abd \ldots\right.}{\sqrt{e+fx} \ldots}}{7b^2df \ldots}$$

$$= -\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+ \ldots}{\ldots}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf \ldots}{105b^2d^2f^3 \ldots}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf \ldots}{105b^2d^2f^3 \ldots}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf \ldots}{105b^2d^2f^3 \ldots}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf \ldots}{105b^2d^2f^3 \ldots}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf \ldots}{105b^2d^2f^3 \ldots}$$

Mathematica [C] time = 13.0155, size = 922, normalized size = 1.2

$$2\left(\sqrt{\frac{bc}{d}-a}\left(\left(C\left(-48d^3e^3+16cd^2fe^2+9c^2df^2e+8c^3f^3\right)+7df\left(5Adf(cf-2de)+B\left(8d^2e^2-3cdfe-2c^2f^2\right)\right)\right)b^3+ \ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (2*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*
C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d
^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9
*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 -
3*c*d*e*f - 2*c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*
(a + b*x)*(c + d*x)*(e + f*x)*(-4*a^2*C*d^2*f^2 + a*b*d*f*(7*B*d*f + C*(-5*
d*e + 2*c*f + 3*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e + c*f + 3*d*f*x))
 + C*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x + 5*f^2*x^
2)))) + I*(b*c - a*d)*f*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*C*f
 - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d^2*
e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 - 3*c
*d*e*f - 2*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sq
rt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt
[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f
)*(4*a^2*C*d^2*f^2 + a*b*d*f*(5*C*d*e + c*C*f - 7*B*d*f) - b^2*(7*d*f*(-4*B
*d*e - 2*B*c*f + 5*A*d*f) + C*(24*d^2*e^2 + 13*c*d*e*f + 8*c^2*f^2)))*(a +
b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x
))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/
(b*c*f - a*d*f)]))/(105*b^4*Sqrt[-a + (b*c)/d]*d^3*f^4*Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.043, size = 9544, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}}{\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}}{\sqrt{fx + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

3.69 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=527

$$-\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)+b^2(-(5df(2Be-3Af)-Ce(cf+8de)))\right.}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (-2*(4*a*C*d*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x])/(15*b^2*d*f^2) + (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e
 + 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellip
ticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e
 - c*C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt
[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSi
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*
f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 0.979913, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)+b^2(-(5df(2Be-3Af)-Ce(cf+8de)))\right.}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a*C*d*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x])/(15*b^2*d*f^2) + (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e
 + 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellip
ticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e
 - c*C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt
[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSi
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*
f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
```

```
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}b(bcCe+3aCde+acCf-5Abdf)-\frac{1}{2}b(4aCdf+b(4Cde+2c\cdots\right.}{\sqrt{a+bx}\sqrt{e+fx}}}{5b^2df}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3\cdots}}{5bdf}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3\cdots}}{5bdf}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3\cdots}}{5bdf}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3\cdots}}{5bdf}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3\cdots}}{5bdf}$$

Mathematica [C] time = 9.69565, size = 562, normalized size = 1.07

$$2\sqrt{a+bx}\left(ibdf\sqrt{a+bx}\sqrt{\frac{bc}{d}-a}(de-cf)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}(-4aCdf+5bBdf-2bC(cf+2de))\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{bc}{d}-\cdots}}{\sqrt{a+b\cdots}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*Sqrt[a + b*x]*((b^2*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B
*d*f) + b^2*(5*d*f*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f
- 2*c^2*f^2)))*(c + d*x)*(e + f*x))/(a + b*x) + b^2*d*f*(c + d*x)*(e + f*x)
*(5*b*B*d*f - 4*a*C*d*f + b*C*(-4*d*e + c*f + 3*d*f*x)) + (I*(b*c - a*d)*f*
(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B*d*f) + b^2*(5*d*f*(-2*
B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2)))*Sqrt[a +
 b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*E
llipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)])/Sqrt[-a + (b*c)/d] + I*b*Sqrt[-a + (b*c)/d]*d*f*(d*e - c*f)*(5
*b*B*d*f - 4*a*C*d*f - 2*b*C*(2*d*e + c*f))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x)
)/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt
[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^4*d
^2*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.032, size = 6049, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}}{\sqrt{bx + a}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{bfx^2 + ae + \left(be + af\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b*f*x
^2 + a*e + (b*e + a*f)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx}\left(A + Bx + Cx^2\right)}{\sqrt{a + bx}\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(e + f*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}}{\sqrt{bx + a}\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

3.70 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=540

$$\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4aCf-3bBf+2bCe)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{f(bc-ad)}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-b}}{}$$

```
[Out] (2*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*Sq
rt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^2*(b*c - a*d)*f*(b*e - a*f))
- (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(
3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sq
rt[d]*f^2*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*S
qrt[-(b*c) + a*d]*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*
Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^
3*Sqrt[d]*f^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.11107, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 154, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{ad}}\right.\right.}{3b^3\sqrt{d}f^2\sqrt{c+dx}(be-af)\sqrt{\frac{b(e+fx)}{be-af}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]),x]
```


```
[Out] (2*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*Sq
rt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^2*(b*c - a*d)*f*(b*e - a*f))
- (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(
3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sq
rt[d]*f^2*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*S
qrt[-(b*c) + a*d]*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*
Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^
3*Sqrt[d]*f^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```


Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
```

```
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{b^2(Bc+2Ad)e+a^2C(3de+cf)-ab(cCe+3Bde\cdots}{2b}\right.}{\cdots}}{(bc\cdots}$$

$$= \frac{2\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \cdots$$

$$= \frac{2\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \cdots$$

$$= \frac{2\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \cdots$$

$$= \frac{2\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \cdots$$

$$= \frac{2\left(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \cdots$$

Mathematica [C] time = 6.81428, size = 551, normalized size = 1.02

$$-\frac{2\left(-ibf(a+bx)^{3/2}(de-cf)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\left(4a^2Cdf-ab(3Bdf+cCf+Cde)+b^2(3Adf+cCe)\right)\text{EllipticF}\left(i\sin\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(-8*a^2*C*d*f^2 + a*b*f*(3*C*d*e + c*C*f + 6*B*
d*f) + b^2*(-3*d*f*(B*e + A*f) + C*e*(2*d*e - c*f)))*(c + d*x)*(e + f*x) +
b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-(b*B) + a*C)
)*f - C*(b*e - a*f)*(a + b*x)) - I*(b*c - a*d)*f*(8*a^2*C*d*f^2 - a*b*f*(3*
C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) + C*e*(-2*d*e + c*f)))*(a
 + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a +
b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*
f)/(b*c*f - a*d*f)] - I*b*f*(d*e - c*f)*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f
) - a*b*(C*d*e + c*C*f + 3*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a
 + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (
b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(3*b^4*Sqrt[-a +
 (b*c)/d]*d*f^2*(b*e - a*f)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.043, size = 4732, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x)
```

```
[Out] 2/3*(4*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1
/2))*a*b^3*c*d*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1
/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+13*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),
((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c*d*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(
1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+3*A*x^2*b^4*d
^2*f^3-3*B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^
(1/2))*a*b^3*c^2*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1
/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+3*B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*
(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-6*B*EllipticE((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*d^2*f^3*(-(f
*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))
^(1/2)+4*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^
(1/2))*a^2*b^2*c^2*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^
(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2)
,((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*d^2*e^3*(-(f*x+e)*b/(a*f-b*e))^(1/2
)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticF((d
*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e^2*f*(-
(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c
))^(1/2)+4*C*x^2*a^2*b^2*d^2*f^3+8*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),
((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^4*d^2*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(
-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-4*C*EllipticF((d*(b
*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*c*d*f^3*(-(f*
x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^
(1/2)+4*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(
1/2))*a^3*b*d^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(
1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),
((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(
1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticF
((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c^2*e*f
^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*
d-b*c))^(1/2)+3*A*b^4*c*d*e*f^2-2*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2
)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+3*B*EllipticF((d
*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c*d*f^3*
(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b
*c))^(1/2)-3*B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*
e))^(1/2))*a^2*b^2*d^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-
b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+3*B*EllipticF((d*(b*x+a)/(a*d-b*c))
^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*d^2*e^2*f*(-(f*x+e)*b/(a*f-b*
e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-3*B*Elli
pticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*
e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/
(a*d-b*c))^(1/2)+6*B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(
a*f-b*e))^(1/2))*a^2*b^2*c*d*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(
a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+9*B*EllipticE((d*(b*x+a)/(a*d-b
*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*e*f^2*(-(f*x+e)*b/(
a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-3*
B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*
b^3*d^2*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*
(b*x+a)/(a*d-b*c))^(1/2)+3*B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*
```

```
c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+
c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-9*C*EllipticE((d*(b*x+a)/
(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*c*d*f^3*(-(f*x+e)*b
/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-
3*A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*
a*b^3*c*d*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*
(b*x+a)/(a*d-b*c))^(1/2)-3*A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*
c)*f/d/(a*f-b*e))^(1/2))*a*b^3*d^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*
x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+3*A*EllipticE((d*(b*x+a
)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*e*f^2*(-(f*x+e)
*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2
)-11*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2
))*a^3*b*d^2*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2
)*(d*(b*x+a)/(a*d-b*c))^(1/2)+C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d
-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*
(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticE((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*e^2*f*(-
(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c
))^(1/2)-9*B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e)
)^(1/2))*a*b^3*c*d*e*f^2*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c)
)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/
2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c*d*e*f^2*(-(f*x+e)*b/(a*f-b*e)
)^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-3*B*a*b^3*
c*d*e*f^2+4*C*a^2*b^2*c*d*e*f^2-C*a*b^3*c*d*e^2*f-C*x^2*b^4*d^2*e^2*f+3*A*x
*b^4*c*d*f^3+3*A*x*b^4*d^2*e*f^2+C*x^3*a*b^3*d^2*f^3-C*x^3*b^4*d^2*e*f^2-3*
B*x^2*a*b^3*d^2*f^3-C*x^2*b^4*c*d*e*f^2-3*B*x*a*b^3*c*d*f^3-3*B*x*a*b^3*d^2
*e*f^2+4*C*x*a^2*b^2*c*d*f^3+4*C*x*a^2*b^2*d^2*e*f^2-C*x*a*b^3*d^2*e^2*f-C*
x*b^4*c*d*e^2*f+C*x^2*a*b^3*c*d*f^3+2*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/
2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*e^3*(-(f*x+e)*b/(a*f-b*e))^(1/2
)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+C*EllipticE((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c^2*f^3*(-
(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c
))^(1/2)+2*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e)
)^(1/2))*a*b^3*d^2*e^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^
(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)+C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e^2*f*(-(f*x+e)*b/(a*f-b*e))^(1/2)*
(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)-2*C*EllipticE((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*e^3*(-(f*x
+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*(d*(b*x+a)/(a*d-b*c))^(
1/2)+3*A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1
/2))*a^2*b^2*d^2*f^3*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1
/2)*(d*(b*x+a)/(a*d-b*c))^(1/2))*(f*x+e)^(1/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/
d/b^4/f^2/(a*f-b*e)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*
x+b*c*e*x+a*c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}}{(bx + a)^{\frac{3}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^2fx^3 + a^2e + \left(b^2e + 2\,abf\right)x^2 + \left(2\,abe + a^2f\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^2*f
*x^3 + a^2*e + (b^2*e + 2*a*b*f)*x^2 + (2*a*b*e + a^2*f)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}}{(bx + a)^{\frac{3}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

3.71 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=597

$$\frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{f(bc-ad)}{d(be-af)}\right)}{3b^3\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}(be-af)}$$

```
[Out] (-2*(4*a^2*C*f + b^2*(3*B*e - 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqr
t[e + f*x])/(3*b^2*(b*e - a*f)^2*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C)
)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/
2)) + (2*Sqrt[d]*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) +
a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)) - b^3*(A*d*e*f +
c*(3*C*e^2 + 3*B*e*f - 2*A*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)^2*Sqrt[c
+ d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(4*a^2*C*d*f + b^2
*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c
 - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[d
]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.35894, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 150, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(-a^2bf(2Bdf+7cCf+13Cde)+8a^3Cdf^2+ab^2(f(-Adf+Bcf+4Bde)+3Ce(4cf+de))-b\right.}{3b^3f\sqrt{c+dx}\sqrt{ad-bc}(be-af)^2\sqrt{\frac{b(e+fx)}{be-af}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a^2*C*f + b^2*(3*B*e - 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqr
t[e + f*x])/(3*b^2*(b*e - a*f)^2*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C)
)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/
2)) + (2*Sqrt[d]*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) -
b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f
) + f*(4*B*d*e + B*c*f - A*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)^2*Sqrt[c
+ d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(4*a^2*C*d*f + b^2
*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c
 - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[d
]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
```

```
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
```

```
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(3Bce-2Acf)-ab(3cCe+3B}{2b}\right.}{\cdots}}{3(b}$$

$$= -\frac{2\left(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-a\right.}{3b(bc-ad)(b}$$

$$= -\frac{2\left(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-a\right.}{3b(bc-ad)(b}$$

$$= -\frac{2\left(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-a\right.}{3b(bc-ad)(b}$$

$$= -\frac{2\left(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-a\right.}{3b(bc-ad)(b}$$

$$= -\frac{2\left(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-a\right.}{3b(bc-ad)(b}$$

Mathematica [C] time = 11.9738, size = 724, normalized size = 1.21

$$-\frac{2\left(b^2f(c+dx)(e+fx)\sqrt{\frac{bc}{d}-a}\left((a+bx)\left(a^2b(2Bdf+4cCf+7Cde)-5a^3Cdf-ab^2(-Adf+Bcf+4Bde+6cCe)+\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C
))*(b*c - a*d)*(b*e - a*f) + (-5*a^3*C*d*f + b^3*(3*B*c*e + A*d*e - 2*A*c*f
) - a*b^2*(6*c*C*e + 4*B*d*e + B*c*f - A*d*f) + a^2*b*(7*C*d*e + 4*c*C*f +
2*B*d*f))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d*f^2 - a
^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B
*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(
c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7
*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2
*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(a + b*x)^(3/2)*Sqrt[
(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*
ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]
 + I*b*(b*c - a*d)*f*(d*e - c*f)*(-4*a^2*C*f + b^2*(-3*B*e + 2*A*f) + a*b*(
6*C*e + B*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
```

```
+ f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^4*Sqrt[-a + (b*c)/d]*(b*c - a*d
)*f*(b*e - a*f)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.094, size = 13614, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{dx + c}}{(bx + a)^{\frac{5}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^3fx^4 + a^3e + \left(b^3e + 3\,ab^2f\right)x^3 + 3\left(ab^2e + a^2bf\right)x^2 + \left(3\,a^2be + a^3f\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^3*f
*x^4 + a^3*e + (b^3*e + 3*a*b^2*f)*x^3 + 3*(a*b^2*e + a^2*b*f)*x^2 + (3*a^2
*b*e + a^3*f)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(5/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{dx + c}}{(bx + a)^{\frac{5}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

3.72 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1034

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{e+fx}(c+dx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-2Bdf)a^3-b^2\left(df(7Bde+2Bcf-3A\cdots\right.\right.}{\cdots}$$

```
[Out] (2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B
*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^(3/2)) - (2*(8*a
^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)
) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 -
13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^2*(b*c - a*d)^2*(b*e
 - a*f)^3*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(d*
f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2))
- a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 - 13
*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
- a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c
*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e
 + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*
e - a*f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 3.15979, antiderivative size = 1034, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1614, 150, 152, 158, 114, 113, 121, 120}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{e+fx}(c+dx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-2Bdf)a^3-b^2\left(df(7Bde+2Bcf-3A\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]),x]
```

```
[Out] (2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B
*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^(3/2)) - (2*(8*a
^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)
) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 -
13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^2*(b*c - a*d)^2*(b*e
 - a*f)^3*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[
```

```
e + f*x])/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(d*
f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2))
- a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 - 13
*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
- a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c
*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e
 + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*
e - a*f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{\sqrt{c+dx}\Big(-\frac{a^2C(3de+cf)+b^2(5Bce-2Ade-4Acf)-ab(5c}{2b}}{}}{}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$= \frac{2\left(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8C\right.}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

Mathematica [C] time = 16.5889, size = 9186, normalized size = 8.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x])
,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.213, size = 33007, normalized size = 31.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{dx + c}}{(bx + a)^{\frac{7}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^4fx^5 + a^4e + (b^4e + 4\,ab^3f)x^4 + 2\,(2\,ab^3e + 3\,a^2b^2f)x^3 + 2\,(3\,a^2b^2e + 2\,a^3bf)x^2 + (4\,a^3be + a^4f)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^4*f
*x^5 + a^4*e + (b^4*e + 4*a*b^3*f)*x^4 + 2*(2*a*b^3*e + 3*a^2*b^2*f)*x^3 +
2*(3*a^2*b^2*e + 2*a^3*b*f)*x^2 + (4*a^3*b*e + a^4*f)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(7/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{dx + c}}{(bx + a)^{\frac{7}{2}}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

3.73 $\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=838

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{5/2}}{7bdf}-\frac{2(2aCdf-b(7Bdf-6C(de+cf)))\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{3/2}}{35bd^2f^2}-\frac{2(5bdf(5bcCe+aC}{}$$

```
[Out] (-2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) + (3*a*d*f - 4*b*(
d*e + c*f))*(2*a*C*d*f - b*(7*B*d*f - 6*C*(d*e + c*f))))*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])/(105*b*d^3*f^3) - (2*(2*a*C*d*f - b*(7*B*d*f - 6*C
*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(35*b*d^2*f^2)
+ (2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - (2*Sqrt[-(b
*c) + a*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) -
(3*b*c*e + a*d*e + a*c*f)*(2*a*C*d*f - b*(7*B*d*f - 6*C*(d*e + c*f)))) + 2*
((a*d*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b
*d*f) + (3*a*d*f - 4*b*(d*e + c*f))*(2*a*C*d*f - b*(7*B*d*f - 6*C*(d*e + c*
f)))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqr
t[-(b*c) + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f
*(3*B*d*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) -
 b^2*(C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f
*(5*A*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]
)/(105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 2.16678, antiderivative size = 831, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{5/2}}{7bdf}+\frac{2(7bBdf-2aCdf-6bC(de+cf))\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{3/2}}{35bd^2f^2}-\frac{2(5bdf(5bcCe+aC}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) - (3*a*d*f - 4*b*(
d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])/(105*b*d^3*f^3) + (2*(7*b*B*d*f - 2*a*C*d*f - 6*b*
C*(d*e + c*f))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(35*b*d^2*f^2)
+ (2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - (2*Sqrt[-(b
*c) + a*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) +
(3*b*c*e + a*d*e + a*c*f)*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))) + 2*
((a*d*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b
*d*f) - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c
*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqr
t[-(b*c) + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f
```

```
*(3*B*d*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) -
 b^2*(C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f
*(5*A*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]
)/(105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{(a+bx)^{3/2}\left(-\frac{1}{2}b(5bcCe+aCde+acCf-7Abdf)+\frac{1}{2}b(7bBdf-2\ldots}{\sqrt{c+dx}\sqrt{e+fx}}}{7b^2df}$$

$$= \frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{5/2}\sqrt{\ldots}}{7b\ldots}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCd\ldots}{105bd^3f^3}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCd\ldots}{105bd^3f^3}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCd\ldots}{105bd^3f^3}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCd\ldots}{105bd^3f^3}$$

$$= -\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCd\ldots}{105bd^3f^3}$$

Mathematica [C] time = 13.8407, size = 1000, normalized size = 1.19

$$\frac{2\left(-\sqrt{\frac{bc}{d}-a}\left(\left(8C\left(6d^3e^3+5cd^2fe^2+5c^2df^2e+6c^3f^3\right)+7df\left(10Adf(de+cf)-B\left(8d^2e^2+7cdfe+8c^2f^2\right)\right)\right)b^3-a\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x])
```

```
,x]
```

```
[Out] (2*(-(b^2*Sqrt[-a + (b*c)/d]*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*f +
 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2) + 7
*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*f +
 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2 +
7*c*d*e*f + 8*c^2*f^2))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f
*(a + b*x)*(c + d*x)*(e + f*x)*(3*a^2*C*d^2*f^2 + 3*a*b*d*f*(14*B*d*f + C*(
-11*d*e - 11*c*f + 8*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e - 4*c*f + 3*
d*f*x)) + C*(24*c^2*f^2 + c*d*f*(23*e - 18*f*x) + 3*d^2*(8*e^2 - 6*e*f*x +
5*f^2*x^2)))) - I*(b*c - a*d)*f*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*
f + 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2)
+ 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*
f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2
 + 7*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x
))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d
]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(3*a
^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(-2*B*d*e - 3*B*c*f + 5*A*d*f)
+ C*(11*d^2*e^2 + 8*c*d*e*f + 16*c^2*f^2)) + b^2*(C*(24*d^3*e^3 + 17*c*d^2*
e^2*f + 16*c^2*d*e*f^2 + 48*c^3*f^3) + 7*d*f*(5*A*d*f*(d*e + 2*c*f) - B*(4*
d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(
a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a +
(b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(105*b^3*Sqrt[-
a + (b*c)/d]*d^4*f^4*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.051, size = 10546, normalized size = 12.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)(bx + a)^{\frac{3}{2}}}{\sqrt{dx + c}\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cbx^3 + (Ca + Bb)x^2 + Aa + (Ba + Ab)x\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{dfx^2 + ce + \left(de + cf\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*b*x^3 + (C*a + B*b)*x^2 + A*a + (B*a + A*b)*x)*sqrt(b*x + a)*sq
rt(d*x + c)*sqrt(f*x + e)/(d*f*x^2 + c*e + (d*e + c*f)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)(bx + a)^{\frac{3}{2}}}{\sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

3.74 $\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=528

$$-\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)-b\left(5df(-3Adf+Bcf+2Bde)-C\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\right)\mathrm{E}\ldots}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (-2*(2*a*C*d*f - b*(5*B*d*f - 4*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]
*Sqrt[e + f*x])/(15*b*d^2*f^2) + (2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e
+ f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a
*c*C*f - 5*A*b*d*f) + (a*d*f - 2*b*(d*e + c*f))*(2*a*C*d*f - b*(5*B*d*f - 4
*C*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
- a*f))])/(15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
) - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) - b*(5*d*f*(2*B*
d*e + B*c*f - 3*A*d*f) - C*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2)))*Sqrt[(b*(c
 + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqr
t[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/
(15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.02759, antiderivative size = 524, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1615, 154, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)F\left(\mathrm{si}\ldots\right.}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(5*b*B*d*f - 2*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b*d^2*f^2) + (2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e +
 f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a*
c*C*f - 5*A*b*d*f) - (a*d*f - 2*b*(d*e + c*f))*(5*b*B*d*f - 2*a*C*d*f - 4*b
*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[Ar
cSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e -
 a*f))])/(15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)])
 - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) + b*C*(8*d^2*e^2
+ 3*c*d*e*f + 4*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(2*d*e + c*f)))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
```

```
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))]])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}b(3bcCe+aCde+acCf-5Abdf)+\frac{1}{2}b(5bBdf-2aCdf-4b\cdots}{\sqrt{c+dx}\sqrt{e+fx}}}{5b^2df}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{\cdots}}{5bdf}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{\cdots}}{5bdf}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{\cdots}}{5bdf}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{\cdots}}{5bdf}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{\cdots}}{5bdf}$$

Mathematica [C] time = 8.05182, size = 615, normalized size = 1.16

$$-\frac{2\left(ibf(a+bx)^{3/2}(bc-ad)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\left(aCdf(cf-de)+5bdf(3Adf-B(2cf+de))+bC\left(8c^2f^2+3cdef+4d^2e^2\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e
 + c*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*
B*(d*e + c*f))))*(c + d*x)*(e + f*x) - b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)
*(c + d*x)*(e + f*x)*(5*b*B*d*f + a*C*d*f + b*C*(-4*d*e - 4*c*f + 3*d*f*x))
 + I*(b*c - a*d)*f*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e + c*f))
- b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*B*(d*e +
c*f))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x)
)/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*
d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a*C*d*f*(-(d*e) + c*f) +
 b*C*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(d*e + 2*c*
f)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(
f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.035, size = 6174, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)\sqrt{bx + a}}{\sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Cx^2 + Bx + A)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{dfx^2 + ce + (de + cf)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(d*f*x
^2 + c*e + (d*e + c*f)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx}\,(A + Bx + Cx^2)}{\sqrt{c + dx}\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}}{\sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

3.75 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=387

$$\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(aCf(de-cf)-b(3df(Be-Af)-Ce(cf+2de)))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{f(bc-ad)}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}} -$$

```
[Out] (2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (2*Sqrt[-(b*c)
+ a*d]*(2*a*C*d*f - b*(3*B*d*f - 2*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*d^(3/2)*f^2*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(a*C*f*(d*e - c*f
) - b*(3*d*f*(B*e - A*f) - C*e*(2*d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*
d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*d^(3/2)*f^2
*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 0.505209, antiderivative size = 384, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1615, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) + (2*Sqrt[-(b*c)
+ a*d]*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*d^(3/2)*f^2*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(B*e - A
*f) - a*C*f*(d*e - c*f) - b*C*e*(2*d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c - a*
d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*d^(3/2)*f^2
*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\int \frac{-\frac{1}{2}b(bcCe+aCde+acCf-3Abdf)+\frac{1}{2}b(3bBdf-2aCdf-2bC(de+cf}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3b^2df}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{(3bBdf-2aCdf-2bC(de+cf))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{3bdf^2} -$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\left((3bdf(Be-Af)-aCf(de-cf)-bCe(2de+cf))\sqrt{\right.}{3bdf^2\sqrt{c+dx}}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}(3bBdf-2aCdf-2bC(de+cf))\sqrt{\frac{b(c+d}{bc-a}}}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{}}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}(3bBdf-2aCdf-2bC(de+cf))\sqrt{\frac{b(c+d}{bc-a}}}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{}}$$

Mathematica [C] time = 5.83552, size = 418, normalized size = 1.08

$$\frac{\sqrt{a+bx}\left(\frac{2ibf\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}\left(aCd(cf-de)+b\left(3Ad^2f+cd(Ce-3Bf)+2c^2Cf\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{bc}{d}-a}}{\sqrt{a+bx}}\right),\frac{bde-adf}{bcf-adf}\right)}{\sqrt{\frac{bc}{d}-a}} - \frac{2b^2(c+dx)(e+fx)(2aCdf}{a+b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a*
C*d*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a +
 (b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqr
t[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[
I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f
)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*d*(C*e
 - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*
x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (
b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f^2*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.028, size = 2497, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 2/3*(C*a*b^2*c*d*e*f+2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))
^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*d^2*f^2+2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)
*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticE((d*(b*
x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c*d*e^2+3*A*(d*(
b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))
^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2
))*a*b^2*d^2*f^2-3*A*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/
2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d
-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c*d*f^2+C*x^2*a*b^2*d^2*f^2+C*x^3*b^3*d^2*f
^2+C*x*a*b^2*c*d*f^2+C*x*a*b^2*d^2*e*f+C*x*b^3*c*d*e*f-3*B*(d*(b*x+a)/(a*d-
b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*Ellip
ticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c*d*e
*f+C*x^2*b^3*c*d*f^2+C*x^2*b^3*d^2*e*f-3*B*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f
*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticE((d*(b*x+a)/
(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b*d^2*f^2+C*(d*(b*x+a
)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2
)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*
b^2*c^2*f^2+2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-
(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)
*f/d/(a*f-b*e))^(1/2))*a*b^2*d^2*e^2-C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e
)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d
-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c^2*e*f-2*C*(d*(b*x+a)/(a
*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*El
lipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c*
d*e^2-2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c
)*b/(a*d-b*c))^(1/2)*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(
a*f-b*e))^(1/2))*a*b^2*c^2*f^2-2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/
(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticE((d*(b*x+a)/(a*d-b*c
))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^2*d^2*e^2+2*C*(d*(b*x+a)/(a*d
-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*Elli
pticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c^2*
e*f-2*C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*
b/(a*d-b*c))^(1/2)*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*
f-b*e))^(1/2))*a*b^2*c*d*e*f-C*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f
-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(
1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b*c*d*f^2+C*(d*(b*x+a)/(a*d-b*c))
^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF(
(d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b*d^2*e*f-
3*B*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a
*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*
e))^(1/2))*a*b^2*d^2*e*f+3*B*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b
*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/
2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^3*c*d*e*f+3*B*(d*(b*x+a)/(a*d-b*c))^(
1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)*EllipticE((d
*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^2*c*d*f^2+3*
B*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d
-b*c))^(1/2)*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e)
)^(1/2))*a*b^2*d^2*e*f)*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^2/b^3/d
^2/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}{bdfx^3+ace+\left(bde+(bc+ad)f\right)x^2+\left(acf+(bc+ad)e\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b*d*f
*x^3 + a*c*e + (b*d*e + (b*c + a*d)*f)*x^2 + (a*c*f + (b*c + a*d)*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

3.76 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=422

$$-\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(aC(de-cf)-b(Adf-Bcf+cCe))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{f(bc-ad)}{d(be-af)}\right)}{b^2\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}}-\frac{2\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2C\right.}{}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (2*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*
e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*
e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqr
t[(b*(e + f*x))/(b*e - a*f)]) - (2*(a*C*(d*e - c*f) - b*(c*C*e - B*c*f + A*
d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Ellip
ticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*
x])
```

Rubi [A] time = 0.69101, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1614, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}f\sqrt{c+dx}\sqrt{ad-bc}(be-af)\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2\sqrt{c+dx}\sqrt{e+f}}{b\sqrt{a+bx}(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (2*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*
e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*
e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqr
t[(b*(e + f*x))/(b*e - a*f)]) - (2*(a*C*(d*e - c*f) - b*(c*C*e - B*c*f + A*
d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Ellip
ticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*
x])
```


Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{-\frac{b^2Bce+a^2C(de+cf)-ab(cCe+Bde+Bcf-Adf)}{2b}+\frac{1}{2}\left(-\frac{2a^2C\cdots}{b}\cdots\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{\cdots}}}{(bc-ad)(be-\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{(aC(de-cf)-b(cCe-Bcf+Adf))\int \frac{\cdots}{\sqrt{\cdots}}}{b(bc-ad)f}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{\left((aC(de-cf)-b(cCe-Bcf+Adf))\sqrt{\frac{b\cdots}{\cdots}}\right.}{b(bc-ad)f\sqrt{\cdots}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+c\cdots\right.}{b^2\sqrt{d}\sqrt{-bc+\cdots}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+c\cdots\right.}{b^2\sqrt{d}\sqrt{-bc+\cdots}}$$

Mathematica [C] time = 5.56614, size = 477, normalized size = 1.13

$$\frac{2\left(\frac{ib(a+bx)^{3/2}(ad-bc)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}(aC(de-cf)+b(Adf-Bde+cCe))\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{bc}{d}-a}}{\sqrt{a+bx}}\right),\frac{bde-adf}{bcf-adf}\right)}{d\sqrt{\frac{bc}{d}-a}}+\frac{b^2(c+dx)(e+fx)\left(2a^2Cdf-ab(Bdf+cCf+Cde)\cdots\right.}{df}\cdots\right.}{b^3\sqrt{a+bx}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*(-(b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)*(e + f*x)) + (b^2*(2*a^2*C*d
*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*(c + d*x)*(e + f*x)
)/(d*f) + (I*(b*c - a*d)*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e +
c*C*f + B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/(Sqrt[-a + (b*c)/d]*d) + (I*b*(-(b*c
) + a*d)*(a*C*(d*e - c*f) + b*(c*C*e - B*d*e + A*d*f))*(a + b*x)^(3/2)*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)
])/(Sqrt[-a + (b*c)/d]*d)))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.045, size = 3984, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 2*(-A*b^4*c*d*e*f-A*x^2*b^4*d^2*f^2+A*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2)
,((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*f^2*(d*(b*x+a)/(a*d-b*c))^(1/
2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-A*EllipticE((d
*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*f^2*
(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b
*c))^(1/2)+B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e)
)^(1/2))*a*b^3*c^2*f^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(
1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(
f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+B*EllipticE((d*(b*x+
a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*d^2*f^2*(d*(b*x+
a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/
2)-C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))
*a^2*b^2*c^2*f^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(
-(d*x+c)*b/(a*d-b*c))^(1/2)+C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b
*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*e^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*
x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-C*EllipticE((d*(b*x+a)
/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c^2*f^2*(d*(b*x+
a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/
2)-C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))
*a^2*b^2*d^2*e^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(
-(d*x+c)*b/(a*d-b*c))^(1/2)-A*x*b^4*c*d*f^2-2*C*EllipticE((d*(b*x+a)/(a*d-b
*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^4*d^2*f^2*(d*(b*x+a)/(a*d-b*c
))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-C*Ellipt
icE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e^
2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d
-b*c))^(1/2)+B*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*
e))^(1/2))*a*b^3*c*d*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))
^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2)
,((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)
*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+C*EllipticF((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c*d*e*f*(d
*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c
))^(1/2)-5*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e)
)^(1/2))*a^2*b^2*c*d*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))
^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-A*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2)
,((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*f^2*(d*(b*x+a)/(a*d-b*c))^(1/2)
*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-A*EllipticF((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*d^2*e*f*(d*(
b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))
^(1/2)+A*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1
/2))*b^4*c*d*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(
-(d*x+c)*b/(a*d-b*c))^(1/2)+A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b
*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*f^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+
e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+3*C*EllipticE((d*(b*x+a)
/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*d^2*e*f*(d*(b*x+a)
/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)
+2*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))
*a*b^3*c^2*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(
d*x+c)*b/(a*d-b*c))^(1/2)+2*C*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b
*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*e^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+
e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+C*EllipticF((d*(b*x+a)/(
a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c^2*e^2*(d*(b*x+a)/(a*
d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+B*x
*a*b^3*c*d*f^2+B*x*a*b^3*d^2*e*f-C*x*a^2*b^2*c*d*f^2-C*x*a^2*b^2*d^2*e*f-A*
x*b^4*d^2*e*f+B*x^2*a*b^3*d^2*f^2-C*x^2*a^2*b^2*d^2*f^2+B*a*b^3*c*d*e*f-C*a
^2*b^2*c*d*e*f+A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-
```

```
b*e))^(1/2))*a*b^3*d^2*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e
))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-A*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/
2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*b^4*c*d*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)
*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-B*EllipticF((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*c*d*f^2*(d
*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c
))^(1/2)-B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^
(1/2))*a^2*b^2*c*d*f^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(
1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)-B*EllipticE((d*(b*x+a)/(a*d-b*c))^(1/2),(
(a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^2*b^2*d^2*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)
*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+C*EllipticF((d*(
b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*c*d*f^2*(d*(
b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))
^(1/2)-C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1
/2))*a^3*b*d^2*e*f*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)
*(-(d*x+c)*b/(a*d-b*c))^(1/2)-2*C*EllipticF((d*(b*x+a)/(a*d-b*c))^(1/2),((a
*d-b*c)*f/d/(a*f-b*e))^(1/2))*a*b^3*c*d*e^2*(d*(b*x+a)/(a*d-b*c))^(1/2)*(-(
f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(1/2)+3*C*EllipticE((d*(b*
x+a)/(a*d-b*c))^(1/2),((a*d-b*c)*f/d/(a*f-b*e))^(1/2))*a^3*b*c*d*f^2*(d*(b*
x+a)/(a*d-b*c))^(1/2)*(-(f*x+e)*b/(a*f-b*e))^(1/2)*(-(d*x+c)*b/(a*d-b*c))^(
1/2))*(f*x+e)^(1/2)*(d*x+c)^(1/2)*(b*x+a)^(1/2)/f/d/(a*f-b*e)/b^3/(a*d-b*c)
/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx + a)^{\frac{3}{2}} \sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^2dfx^4 + a^2ce + \left(b^2de + \left(b^2c + 2\,abd\right)f\right)x^3 + \left(\left(b^2c + 2\,abd\right)e + \left(2\,abc + a^2d\right)f\right)x^2 + \left(a^2cf + \left(2\,abc + a^2d\right)e\right)x},$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^2*d
*f*x^4 + a^2*c*e + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*x^3 + ((b^2*c + 2*a*b*d)
*e + (2*a*b*c + a^2*d)*f)*x^2 + (a^2*c*f + (2*a*b*c + a^2*d)*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx + a)^{\frac{3}{2}}\sqrt{dx + c}\sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

3.77 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=642

$$-\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(a^2Cd(de-cf)+ab\left(3f\left(Ad^2+c^2C\right)-Bd(2cf+de)\right)-b^2\left(Acdf+2Ad^2e-3Bcde+3c^2Ce\right)\right)\text{Ellip}}{3b^2\sqrt{d}\sqrt{c+dx}\sqrt{e+fx}(ad-bc)^{3/2}(be-af)}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(3/2)) + (2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*
c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt
[a + b*x]) - (2*Sqrt[d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A
*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqr
t[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*(
-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a
*f)]) - (2*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e + 2*A*d^2*e +
A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*Sqrt[(b*(c + d*x)
)/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*
Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] time = 1.51666, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 152, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(a^2Cd(de-cf)+ab\left(3f\left(Ad^2+c^2C\right)-Bd(2cf+de)\right)-b^2\left(Acdf+2Ad^2e-3Bcde+3c^2Ce\right)\right)F\left(\sin\right.}{3b^2\sqrt{d}\sqrt{c+dx}\sqrt{e+fx}(ad-bc)^{3/2}(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```


```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(3/2)) + (2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*
c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt
[a + b*x]) - (2*Sqrt[d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A
*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqr
t[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*(
-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a
*f)]) - (2*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e + 2*A*d^2*e +
A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*Sqrt[(b*(c + d*x)
)/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*
Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```


Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
```

```
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
```

```
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{-\frac{a^2C(de+cf)-ab(3cCe+Bde+Bcf-3Adf)+b^2(3Bce-2A(de+}{2b}}{(a+bx)^{3/}}}{3(bc-}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4A\right.}{}$$

Mathematica [C] time = 10.9311, size = 699, normalized size = 1.09

$$-\frac{2\left(b^2(c+dx)(e+fx)\sqrt{\frac{bc}{d}-a}\left((a+bx)\left(a^2b(4C(cf+de)-Bdf)-2a^3Cdf-ab^2(-4Adf+Bcf+Bde+6cCe)+b^3(3B\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C))
*(b*c - a*d)*(b*e - a*f) + (-2*a^3*C*d*f - a*b^2*(6*c*C*e + B*d*e + B*c*f -
 4*A*d*f) + b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(-(B*d*f) + 4*C*(d*e +
c*f)))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(2*a^3*C*d*f + a*b^2*
(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d*e + c*f)) + a^
2*b*(B*d*f - 4*C*(d*e + c*f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(2*a^3
*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d
*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[
Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(
b*c - a*d)*(a^2*C*f*(d*e - c*f) + b^2*(3*c*C*e^2 + A*d*e*f + c*f*(-3*B*e +
2*A*f)) + a*b*(-3*C*d*e^2 + f*(2*B*d*e + B*c*f - 3*A*d*f)))*(a + b*x)^(3/2)
```

```
*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipt
icF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a
*d*f)])))/(3*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^(
3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] time = 0.122, size = 12981, normalized size = 20.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx + a)^{\frac{5}{2}} \sqrt{dx + c} \sqrt{fx + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}}{b^3dfx^5 + a^3ce + \left(b^3de + \left(b^3c + 3\,ab^2d\right)f\right)x^4 + \left(\left(b^3c + 3\,ab^2d\right)e + 3\left(ab^2c + a^2bd\right)f\right)x^3 + \left(3\left(ab^2c + a^2bd\right)e +\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^3*d
*f*x^5 + a^3*c*e + (b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*x^4 + ((b^3*c + 3*a*b^
2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*x^3 + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b
*c + a^3*d)*f)*x^2 + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

3.78 $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1116

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(bc-ad)(be-af)(a+bx)^{5/2}}+\frac{2\sqrt{d}\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf))a^3-b^2\left(C\left(3d^2e^2-13cdfe+3\right.\right.\right.}{}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(5/2)) + (2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e + B
*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*C*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b*(b*c - a*d)^2*(b*e - a*f)^2*
(a + b*x)^(3/2)) + (2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(
10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c
^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
15*b*(b*c - a*d)^3*(b*e - a*f)^3*Sqrt[a + b*x]) + (2*Sqrt[d]*(2*a^4*C*d^2*f
^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B
*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e -
23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a
^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e +
 c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])
/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]) + (2*Sqrt[d]*(a^3*C*d*f*(d*e - c*f) + b^3*(8*A*d^2*e^2 - c*
d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2)) + a*b^2*(d^2*e*(
2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2 - 27*B*e*f + 11*A*f^
2)) - 3*a^2*b*(d*f*(2*B*d*e + 3*B*c*f - 5*A*d*f) - C*(d^2*e^2 + c*d*e*f + 3
*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rubi [A] time = 3.34186, antiderivative size = 1116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1614, 152, 158, 114, 113, 121, 120}

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(bc-ad)(be-af)(a+bx)^{5/2}}+\frac{2\sqrt{d}\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf))a^3-b^2\left(C\left(3d^2e^2-13cdfe+3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```



```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(5/2)) + (2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e + B
*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*C*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b*(b*c - a*d)^2*(b*e - a*f)^2*
(a + b*x)^(3/2)) + (2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(
```

```
10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c
^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f)))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
15*b*(b*c - a*d)^3*(b*e - a*f)^3*Sqrt[a + b*x]) + (2*Sqrt[d]*(2*a^4*C*d^2*f
^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B
*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e -
23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a
^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e +
 c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])
/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]) + (2*Sqrt[d]*(a^3*C*d*f*(d*e - c*f) + b^3*(8*A*d^2*e^2 - c*
d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2)) + a*b^2*(d^2*e*(
2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2 - 27*B*e*f + 11*A*f^
2)) - 3*a^2*b*(d*f*(2*B*d*e + 3*B*c*f - 5*A*d*f) - C*(d^2*e^2 + c*d*e*f + 3
*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{-\frac{a^2C(de+cf)-ab(5cCe+Bde+Bcf-5Adf)+b^2(5Bce-4\cdots}{2b}}{(a\cdots}}{5(b\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} + \frac{2\left(2a^3Cdf+ab^2(10cCe+Bde+Bcf\cdots\right.}{\cdots}$$

Mathematica [C] time = 16.5791, size = 8844, normalized size = 7.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.297, size = 34102, normalized size = 30.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx + a)^{\frac{7}{2}}\sqrt{dx + c}\sqrt{fx + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{bx + }}{b^4dfx^6 + a^4ce + \left(b^4de + \left(b^4c + 4\,ab^3d\right)f\right)x^5 + \left(\left(b^4c + 4\,ab^3d\right)e + 2\left(2\,ab^3c + 3\,a^2b^2d\right)f\right)x^4 + 2\left(\left(2\,ab^3c + 3\,a^2b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)/(b^4*d
*f*x^6 + a^4*c*e + (b^4*d*e + (b^4*c + 4*a*b^3*d)*f)*x^5 + ((b^4*c + 4*a*b^
3*d)*e + 2*(2*a*b^3*c + 3*a^2*b^2*d)*f)*x^4 + 2*((2*a*b^3*c + 3*a^2*b^2*d)*
e + (3*a^2*b^2*c + 2*a^3*b*d)*f)*x^3 + (2*(3*a^2*b^2*c + 2*a^3*b*d)*e + (4*
a^3*b*c + a^4*d)*f)*x^2 + (a^4*c*f + (4*a^3*b*c + a^4*d)*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```